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PHILOSOPHICAL ESSAYS  
FOR  
ALFRED NORTH WHITEHEAD



PHILOSOPHICAL ESSAYS  
*for*  
ALFRED NORTH WHITEHEAD

*February Fifteenth*  
*Nineteen Hundred and Thirty-Six*

NEW YORK / RUSSELL & RUSSELL

FIRST PUBLISHED IN 1936  
REISSUED, 1967, BY RUSSELL & RUSSELL  
A DIVISION OF ATHENEUM HOUSE, INC.  
L. C. CATALOG CARD NO: 66-24769



## FOREWORD

THE following essays were written for Alfred North Whitehead by a group of his students. The membership of this, as of any such group, is arbitrary. It might have been either larger or smaller. The individuals composing the group, however, represent many different tendencies in contemporary American philosophy, and the volume indicates some, at least, of the many directions in which the thought of Whitehead is being felt.

The arrangement of the essays calls for a word. Three are distinctly more historical in approach than the others. They have been placed first, and are followed by the remaining ones, arranged according to their subjects, which fall roughly under the headings of logic and methodology, metaphysics and ethics.

The essays express the interest of their writers. Each deals with one special topic, and at the same time suggests the general philosophical viewpoint of its author. Consequently, the content of the volume is extremely varied. Nevertheless, certain leading ideas appear in many of the papers, though in very different forms and contexts, and thus relate the papers both to each other and to the philosophy which has influenced them all. Whether there is anything of the spirit of that philosophy in them — and the hope that there is constitutes the sole excuse for the appearance of this volume — the reader must judge.



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# THE MATHEMATICAL BACKGROUND AND CONTENT OF GREEK PHILOSOPHY

BY F. S. C. NORTHROP

THE foundations of modern mathematics were laid by Georg Cantor. His work led to the systematic derivation of mathematics from logic which was accomplished by Russell and Whitehead in *Principia Mathematica*. This required an entirely new theory of logic. The new logic in turn had a profound and far-reaching effect upon philosophy.

It is now known that Cantor's theory leads to contradictions. Moreover, these contradictions are so fundamental that they appear in the logic as well as the technical mathematics. The significance of this for contemporary logic, mathematical science and philosophy should be obvious: Traditional modern theories of these subjects cannot be trusted. New mathematical, logical, and philosophical theory must be constructed.

The proper way to determine the character of this new theory seems evident. Cantor's basic assumption, upon which modern mathematical theory rests, should be examined to determine the fundamental problem with reference to which his theory is a particular answer. Once this problem is known we should be able to designate the other possible answers which it permits and thereby gain a clue to a possible way out of our present difficulties.

A reading of Cantor's original papers will show that his basic assumption is the existence of an actual infinite. It will indicate also that he regarded his mathematical ideas not merely as providing a new and rigorously formulated theory of mathematics but also as presupposing and requiring an original answer to the metaphysical problem of the one and

the many.<sup>1</sup> It appears, therefore, that the fundamental problem which we face in logic, mathematics and philosophy at the present moment is, at bottom, none other than this old metaphysical issue.

If we are to avoid falling into old errors in our present attempts to provide it with a solution, it seems to be essential that we should know what previous answers to this problem were given, what mathematical considerations led to their origin, and what later mathematical discoveries caused their rejection. These alternatives to the modern theory of the foundations of mathematics are to be found in great part in the Elements of Euclid. In order to get them before us, it is necessary to arrange the thirteen books of Euclid in the order of their temporal origin. When this is done and the Greek philosophers are located in their proper temporal positions with reference to this mathematical background, it happens that considerable new light is thrown upon the content and development of Greek philosophy.

Several centuries of scientific investigation are at the basis of Greek philosophy. Unless the science is known, the philosophical theories are devoid of content and easily misunderstood, and the source of their authority for classical and medieval minds is completely missed.

Mathematics and astronomy were the mature and leading sciences in the Greek period. Furthermore, mathematics was considered as a natural science. The modern conception of it as a subjectively-created subject which deals only with the possibles had not arisen. Instead, it was regarded as the basis of the actual and the necessary in nature. This conception was most natural. The original notions in mathematics were magnitude and number. They had their basis and origin in the observed continuous extension of nature and the many diverse things within this one continuum. The continuity

<sup>1</sup> Georg Cantor. *Gesammelte Abhandlungen*. 1932. Especially pp. 370-377.

of nature gave birth to geometry ; its diversity brought forth arithmetic. In reaching a theory of the relation between these two factors, the fundamental mathematical problem concerning the relation between geometry and arithmetic, and the corresponding and more general fundamental metaphysical problem of the one and the many, arose.

Anaximander was the first to explicitly designate nature as a single continuum. He called it the "Boundless." However, its first scientific formulation appeared when Anaxagoras defined it in terms of the principle that "there is a smaller but never a smallest."<sup>2</sup> This denies the existence of extended segments or atoms which are not further divisible, and clearly puts the conception of nature as irreducibly atomic into opposition with the conception of it as a real continuum.

The Anaxagorean theory had its difficulties. Strictly speaking, it provided mathematics with but one actual entity — namely, extended nature as a whole. But a science of mathematics is impossible unless there are elements upon which to operate. To meet this demand the infinitely divisible continuum was conceived as made up, in any finite segment, of an infinite number of points. An examination of the first two definitions of Book I of Euclid will show that Greek mathematics began with this conception.

But Zeno raised the question : Do the points have extension, or are they without magnitude? He had no difficulty in showing that either assumption reveals a contradiction in this first theory of the foundations of mathematics. If the points have extension then there can be only a finite number of them in a finite magnitude and the Anaxagorean principle that any magnitude is divisible without limit is invalid. If, on the other hand, the points have no extension, then even an infinite number of them will give no magnitude whatever, and the assumption that nature is an extended continuum is contradicted. This first instance of the demonstration of a contra-

<sup>2</sup> E. Frank. *Plato und die Sogenannten Pythagoreer*. Halle. 1923. p. 47.

diction in the foundations of mathematics constituted a "veritable scandal" in Greek science.<sup>3</sup>

The first attempt at a resolution of the paradox was made by Democritos. He drew the obvious conclusion. Since nature is extended, and this cannot be the case if it be constituted, in any segment, by an infinite number of points, it follows that there must be an end to division and any finite interval must be constituted of a finite number of extended indivisible points. Luria has recently shown, in an exceedingly important article,<sup>4</sup> that the atoms of Democritos were arithmetical, as well as physical, units and that his atomic theory arose as much to meet Zeno's attack upon the traditional theory of the foundations of mathematics, as it did to meet Parmenides' arguments concerning the invalidity of the traditional concept of motion in physics.

The evidence in support of this first attempt to reduce geometry to arithmetic is well known.<sup>5</sup> The Pythagorean study of music and of triangular, square, and rectilinear figures and numbers supported it, as did Democritos' important work in founding the experimental and mathematical science of acoustics.<sup>6</sup> The theory worked beautifully except for the discovery of incommensurable magnitudes. Give the atomic arithmetical unit any fixed magnitude one pleases; — if it goes into the side of a square a definite number of times leaving nothing over, it will not do so in the diagonal. This discovery of what we term the irrational, produced the second revolution in the fundamental concepts of Greek mathematics. It made a tremendous impression upon the Greek

<sup>3</sup> H. Hasse u. H. Scholz. *Die Grundlagenkrise der Griechischen Mathematik.* pp. 10-12; P. Tannery. *Pour l'Histoire de la Science Hellene.* pp. 247-61.

<sup>4</sup> S. Luria. *Die Infinitesimal Theorie der Antiken Atomisten.* Quellen u. Studien zur Geschichte der Mathematik Astronomie und Physik. Abt. B. (Hereafter referred to as Q. u. S.) Bd. 2. pp. 106-185.

<sup>5</sup> H. G. Zeuthen. *Die Mathematik im Alterum und im Mittelalter.* Die Math. Wissensch. III. 1. 31, 37, 40. Also Burnet. *Greek Philosophy Part I.* A. Reymond. *History of the Sciences in Greco-Roman Antiquity* pp. 120-125.

<sup>6</sup> See E. Frank. pp. 150-181.

mind.<sup>7</sup> Plato says in one of his dialogues that anyone who does not know about it is "guilty of a swinish ignorance."<sup>8</sup> It had a profound and lasting effect upon philosophy, as we shall see. Eventually it convinced Greek mathematicians that geometry cannot be reduced to arithmetic, and, thereby, persuaded philosophers that the metaphysical problem of the one and the many must be answered in terms of the primacy of the one.

But before this conclusion was accepted, attempts were made to reconcile the discovery of incommensurables with the arithmetical theory of indivisible extended atomic units. As Luria has shown, Democritus had a reasonable answer. A distinction must be drawn, he pointed out, between what scientific theory conceives nature to be, and what to our feeble senses, it appears to be. Nature is constituted of indivisible arithmetical and physical units with extension, but these units are so small that our senses cannot discriminate them. Hence, what is a discontinuous association of atoms, appears to us as a continuum. Nature as known by this atomic scientific theory gives "genuine knowledge"; nature as given through our feeble senses gives what Democritus termed "spurious knowledge." In the real world of atoms everything is expressible in terms of numbers and ratios between them, and there are no incommensurable magnitudes; the source of incommensurability is solely in the world of sensation.

This Democritean distinction between the real world as conceived by the intellect in terms of scientific theory, and the apparent world as given by the senses, passed over through Archytas and other later "so-called Pythagoreans"<sup>9</sup> to Plato. The influence upon philosophy of the discovery of the irrational is tremendous.

<sup>7</sup> P. Tannery. *La Géométrie Grecque*. p. 98.

<sup>8</sup> *Laws* 819 d.

<sup>9</sup> See Aristotle. *Metaphysics* I 985b, and E. Frank. *Plato und die Segenannten Pythagoreer*. 1923.

For Democritos and the later Pythagoreans, and apparently for Plato also, in the *Philebos*, it provided a scientific criterion for the distinction between the rational and the non-rational. Since the real world as conceived by scientific theory contains no incommensurables and is constituted completely by atomic arithmetic units, it can be expressed in terms of ratios. Such a world could be called a rational world. On the other hand, because the world of sensation is the source of incommensurables, it cannot be defined in terms of ratios; hence, it is irrational.<sup>10</sup>

Furthermore, since the real world of indivisible elements defined by scientific theory puts an end to division and is definite, it was called the world of being, and was designated later by Plato, in the *Philebos*, as "the class of the limit." Likewise the intuitively-given continuum of Democritos, since it is divisible without limit, and hence is indeterminate, is called a world of becoming, and is designated by Plato as "the class of the infinite," and the "indeterminate dyad."<sup>11</sup> Even the epistemological distinctions in Greek philosophy have a mathematical content and basis.

The Democritean distinction between the intuitively-given sense continuum and the conceptually-grasped world of indivisible extended atoms was equally important for mathematics. Luria has shown<sup>12</sup> that Democritos used this distinction to suggest the mathematical technique of treating geometrical continua which have non-rectilinear boundaries, and hence are unanalyzable directly, as limiting cases of a

<sup>10</sup>*Philebos* 25 reads "the class of the limit . . . by introducing number makes the differing elements commensurable and harmonious." When we consider Plato's theory of ideal numbers we shall see that this means something different for Plato from what it would have meant for Democritos. Nevertheless, upon the fundamental point that the criterion for the distinction between the rational and the irrational is expressibility in terms of ratios, Plato and Democritos are in agreement.

<sup>11</sup> See L. Robin. *La Théorie Platonicienne des Idées et des Nombres*. 1908. p. 444.

<sup>12</sup> *Q. u. S. Bd. 2.* pp. 129-148.

series of rectilinear figures of a quite different kind with determinate geometrical properties which can be directly analyzed. This, as Luria indicates, is the origin of the method of exhaustion and the infinitesimal calculus. Archimedes informs us that Democritos actually used it to discover and roughly prove the theorem concerning the ratio between the volume of a cone and that of a cylinder of the same height and base, which Eudoxos later proved rigorously.<sup>13</sup>

Nevertheless the Democritean theory of the foundations of mathematics was inadequate. By restricting science to the atomic world of commensurable magnitudes which can be expressed in terms of numerical ratios, it left geometrical figures with incommensurable sides outside of science. Moreover, the solution of the difficulty was exceedingly artificial. If the side of a square is a permissible scientific concept, then the diagonal would seem to be so also. Yet the Democritean theory placed one in the real world and the other in the sense world. Certainly the footing of one in reality is as good as that of the other, especially since a different choice of the unit length will reverse the status of each.

Obviously, it was necessary to construct a theory of mathematics which would provide for both types of magnitude. This, the crowning achievement of Greek mathematics, was accomplished by Eudoxos in his general theory of proportions, given in the Fifth Book of Euclid's Elements.

The Pythagoreans had constructed a theory of proportions for commensurable magnitudes; the Seventh Book of Euclid's Elements is restricted to it. The new Eudoxian theory of proportions applied to any magnitude or quantity whatever; thus arithmetic and geometry, including commensurable and incommensurable magnitudes, were brought under a single theory. The fundamental idea in it appears in the Fifth Defi-

<sup>13</sup> The Thirteen Books of Euclid's Elements, T. L. Heath. (Hereafter referred to as Euclid's Elements, T. L. Heath.) II. p. 40 and III. pp. 366-368.

inition of Book V of Euclid, concerning magnitudes which are "said to be in the same ratio."<sup>14</sup>

The essential point to be noted is that it made the concept of ratio, rather than the concept of number, primary in mathematical science up to the modern world. With Eudoxos, the Democritean and Pythagorean attempt to reduce geometry to arithmetic by introducing indivisibles with extension, failed. This conclusion is conclusively confirmed by the fact, emphasized by the mathematical historian, M. Cantor,<sup>15</sup> that to the very end Greek mathematicians conceived of numbers merely as the natural numbers.<sup>16</sup> This restricted the applicability of arithmetic solely to commensurable magnitudes and made the reduction of geometry to arithmetic impossible. The concept of number was not generalized to include zero, fractions and irrationals. The Greek theory becomes clear if one realizes that, instead of generalizing the notion of number, as did the moderns, the Greeks gained a unified theory of geometry and arithmetic by generalizing the concept of ratio. The equivalents of fractions and irrationals existed for them, but they were not numbers. Instead fractions were ratios between natural numbers, and irrationals were ratios between magnitudes.

It was not an accident that the Greek word for ratio in the crucial definition<sup>17</sup> in the Fifth Book of Euclid's Elements is the same word *λόγος* which designates the rational principle in Plato's philosophy and the divine principle in the Fourth Gospel. When informed men in the western world, from Plato and Aristotle on to Augustine and St. Thomas, regarded

<sup>14</sup> See Euclid's Elements. T. L. Heath. II. pp. 112-126.

<sup>15</sup> Vorlesungen über Geschichte der Mathematik. 1880. Vol. I. p. 159; Dritte Auflage. 1907. pp. 187-8. See also O. Toeplitz. Q. u. S. Ab. B. Vol. I. 29-33, and H. Hasse u. H. Scholz. Die Grundlängenkrise der Griechischen Mathematik. Charlottenburg. 1928. Especially Anhang pp. 34-72.

<sup>16</sup> This makes it exceedingly difficult to accept A. E. Taylor's theory (Philosophical Studies. 1934. Ch. III.) that Plato had the modern theory of the irrational number.

<sup>17</sup> Def. 3. Euclid V. See Euclid's Elements. T. L. Heath. II. p. 116.

their universe and the potentialities of reason in themselves as grounded in a rational principle which they termed the *lóγος* they were but giving expression to the accepted and crowning conclusion of Greek mathematics,—the conclusion, namely, that ratio is an irreducible and primary concept.

Their confidence in the existence of a rational principle of unity had precisely the same scientific foundations. An irreducible concept of ratio suggests the primacy of the one, since ratio emphasizes relatedness rather than relata.<sup>18</sup> But the Eudoxian general theory of proportions made the triumph of monism over pluralism explicit. For the Eudoxian theory is not valid unless the Anaxagorean principle that “there is a smaller but never a smallest” is accepted.<sup>19</sup> This principle is incompatible with the conception of nature as constituted of an irreducible atomic many. The one continuum becomes primary, and numbers and the many exist potentially but not actually. Proposition I in Book X, and the lemma at the beginning of Book XII of Euclid embody this Anaxagorean principle.

Aristotle did not miss this point. Coming after Eudoxos, he saw that mathematics invalidated the Democritean theory of arithmetical atoms, and the Platonic theory of elemental geometrical figures. In *de Caelo* III 4, 303a20 he writes: “Because they (Leucippos and Democritos) speak of indivisible substances they must necessarily be in conflict with mathematics.” A comment by Simplicius on *de Caelo* III 7, 306a26 adds that “those who distinguish the substances by their forms, as the fire as pyramid, the water as octohedron (Platonists) or those who hold the substances to be indivisible are required to hold that not all bodies are divisible and to come in conflict with mathematics, in that they maintain that which the science opposes.”<sup>20</sup> In other words, the Eudoxian general theory of proportions in conjunction with the method

<sup>18</sup> Def. 3. Euclid Bk. V reads “a ratio is a sort of relation. . .”

<sup>19</sup> See E. Frank. p. 60; Reymond. p. 133.

<sup>20</sup> See also *De Gen. et. Cor.* I 325 b 25.

of exhaustion which it made possible necessitates that nature be regarded as one irreducible continuum. Plurality enters merely through the qualities that differentiate the one continuum rather than through the existence of an actual many.

The science of astronomy provided another source for the authority of the principle of rational unity in the Greek and medieval world. This science was especially important for Plato, since he came, at least in his thinking, between Democritus and Eudoxos, and probably did not attain the complete Eudoxian theory of proportions.<sup>21</sup>

However, it was not necessary to wait until Eudoxos to learn of the significance of the concept of ratio. This concept was discovered in music by the Pythagoreans and applied to acoustics by Democritus. All—Democritus, Archytas, and Plato alike—generalized it to apply to the universe as a whole.<sup>22</sup>

The detailed steps in this scientific generalization were carried out by Theaetetus, among others. Beginning with the regular polygons in Book IV of Euclid, which Plato, in the *Timaeos*, divides into the elemental atomic triangles, Theaetetus constructs the five regular solids and carries through the scientific development which culminates in Book XIII of Euclid's *Elements*.

Theaetetus also demonstrated that there are but five regular solids in a three-dimensional area.<sup>23</sup> This suggested that the three-dimensional universe and all the things within it must be made up of five basic materials. This caused them to look to the sense world for the five perceivable correlates of the five conceived regular solids. The empirical chemistry of the time had reduced all perceived things to earth, air, fire and

<sup>21</sup> Our reasons for differing with A. E. Taylor (*Philosophical Studies*, p. 97) and O. Toeplitz (*Q. u. S. I.* pp. 3-33) upon this point will be indicated later.

<sup>22</sup> E. Frank. pp. 150-184.

<sup>23</sup> See Eva Sachs. *Die Fünf Platonischen Körper*. *Philologische Untersuchungen*. 1917. Heft. 24. pp. 1-242.

water.<sup>24</sup> Hence in these and the ether or the "cosmic body" they found a verification of their mathematical theory. The five regular solids in the real world, known with the mathematical intellect, were correlated with perceived earth, air, fire, water and the ether given in the world of sensation, much as Einstein recently correlated the ten  $g_{ik}$  required by an invariant equation for a four-dimensional world with the ten observable or measurable potentials of the gravitational field.

Observation also indicated to the Greeks that perceived earth, air, fire and water pass over into one another.<sup>25</sup> Hence they must be complex rather than simple substances. This caused Plato, or some scientist from whom Plato received the conception recorded in the *Timaeos* 53D, to analyze the five regular solids into common atomic geometrical plane figures out of which they could be constructed. He found these by taking the three regular plane figures, given in Book IV of Euclid, out of which the five regular solids can be constructed, and by dividing these three plane figures into smaller triangular figures. It can be shown that all can be constructed out of four elemental atomic triangles.<sup>26</sup> These atomic triangles were the elemental "mathematicals"<sup>27</sup> of Plato's philosophy.

Out of them, everything in the universe could be constructed. They were first combined into the equilateral triangle, the square, and the regular pentagon.<sup>28</sup> These were then combined to produce the five regular solids, identified with earth, air, fire, water, etc., and the latter, in turn, were then combined to produce everything in the universe including man, as well as stone and cloud and waterfall. Thus the science of stereometry provided the Greeks with a mathe-

<sup>24</sup> *Timaeos* 53D. A. E. Taylor's Commentary on Plato's *Timaeos* should be consulted in connection with this dialogue.

<sup>25</sup> *Timaeos* 49B.

<sup>26</sup> See Euclid's Elements. T. L. Heath. II. 98, and *Timaeos* 54.

<sup>27</sup> See Aristotle. *Metaphysics*. I. 987 b 15, and 992 b 15 (W. Christ's trans.).

<sup>28</sup> See Euclid's Elements. T. L. Heath. II. 98.

mathematical physical chemistry<sup>29</sup> which was able to draw a distinction between the molecular regular solids and the atomic plane triangles. Thereby the mathematical theory of nature was revealed as applying, not merely generally, but in detail, to every concrete object and process in the universe. This is the reason why Plato prescribes in the Republic that after studying the foundations of mathematics, the philosopher and the statesman must pass on to plane geometry and stereometry. The knowledge of fundamentals would provide the few elemental concepts or "ideas" in terms of which everything must be defined if it is to be understood; the science of plane and solid geometry would indicate the theory of the compounding of these basic ideas and provide the basis for their detailed application in the concrete world. Readers of the Republic quite underestimate the importance of Plato's education for the philosopher king when they regard it as a mere utopian dream. It represented the actual sequence of scientific ideas in Plato's time.

The fact that he prescribes this training for philosophers and statesmen and moralists as well as for natural scientists indicates also that he saw that the source of authority for political and cultural theory is to be found in the mathematical natural sciences. It was because he perceived that a dependable theory of the state rests upon a dependable philosophy and that a dependable philosophy which can command free assent and agreement must rest upon the objectively-determined concepts of the natural sciences which are independent of the sophistical variations in human opinion, that he said the statesman must be a philosopher and prescribed that the philosopher must arrive at the intellectual outlook which defines his cultural "idea of the good" through the study of arithmetic, stereometry and astronomy.

An adequate understanding of Plato's theory of ideas is impossible without a knowledge of his mathematical theory of "ideal numbers." The latter theory arose as a result of an

<sup>29</sup> See A. N. Whitehead. *Process and Reality*. pp. 144-5.

attempt to overcome the difficulties, previously mentioned, in the Democritean theory, which Plato took over and modified. Instead of treating the individual atoms merely as arithmetical units, Plato regarded them as atomic surfaces; i.e., the atomic triangles mentioned earlier. An examination of the sides of these atomic triangles will reveal that they are incommensurable.<sup>30</sup> This permitted Plato to bring incommensurable magnitudes within the atomic world of "genuine" knowledge which science treats. This also had the advantage of accounting for the importance of stereometry by making geometrical form an atomic idea rather than a complex notion arising from the chance aggregating of merely arithmetical physical units.

But having done this, one other unusual consequence followed. The atomic triangles are the smallest geometrical figures. They are the elementary constituents of the universe. Nevertheless, they are definable by the ratios of the lengths of their sides. Hence the ratios which define these atomic triangles are irreducible elementary "ideas."<sup>31</sup> The fact that there are but four atomic triangles limits these ratios to a small finite number. But these ratios relate sides which are incommensurable.<sup>32</sup> Hence this small finite number of elementary ratios cannot be defined in terms of arithmetical natural numbers. Nevertheless, they are in the conceived world of scientific theory. Hence Plato calls them "ideas." This small finite number of elemental ratios, considered in and by themselves, apart from the somewhat material atomic triangles which they define and constitute when they "mix" with the "indeterminate dyad," are the elemental "ideas" of Plato's philosophy. When these purely conceived, ideal ratios merge with the undifferentiated intuitively-given continuum, the first determinate objects to be "generated" are the atomic

<sup>30</sup> Timaeos. 53c-55n. Epinomis. 99od. A. E. Taylor. Forms and Numbers. Philosophical Studies. 1934. p. 116.

<sup>31</sup> See Euclid's Elements. T. L. Heath. I. p. 8 referring to "the conception or definition (*λόγος*) of figure."

<sup>32</sup> Philebos. 25e 26.

triangles. The intuitively-given "Boundless" or "class of the infinite" contributes the matter, the elemental ideal ratios contribute the form.

In this distinction between the pure ratios and the atomic triangles the Platonic distinction between "ideas" and "mathematicals" consists. The necessity for this distinction is demonstrated by the presence of similar triangles of great and small sizes; the triangles or "mathematicals" are many and different, the ratios which define them are one and the same. This shows that the triangles have a little matter in them, and is another reason for regarding the pure ratios as "ideas."

When the elementary ratios compound, the molecular "mathematicals" or regular solids arise, and when these in turn "mix" with the intuitively-given "Boundless," the "sensibles" such as perceived earth, perceived air, perceived fire and perceived water appear. As the complications in the combinations of the ideal ratios and the "mathematicals" increase, the corresponding differentiations in the sense world increase. This increase in the differentiations of the sense world from the original one undifferentiated "Boundless" with which the process of generation begins, gives rise,<sup>33</sup> in a peculiar order, to the series of natural numbers.<sup>34</sup> Thus Plato's distinction between the "ideal numbers" and the "natural numbers" arises. The "ideal numbers" are purely in the world of "genuine knowledge" known only to the mathematical intellect. The "natural numbers" are in the "mixed world" which results from the first merging of the "ideal numbers"

<sup>33</sup> A detailed account of this process involves an understanding of (1) The Pythagorean notion of the "logos-cutter," (2) the Platonic *διαίρεσις* and (3) the relation of the atomic ratios to each other. For (1) see E. R. Goodenough. A Neo-Pythagorean Source in Philo Judaeus. Yale Classical Studies Vol. 3. For different theories of (2) see J. Stenzel, *Zahl und Gestalt bei Platon u. Aristoteles*, 1933; O. Becker, *Q. u. S. I.* pp. 464-501; L. Robin, *La Theorie Platonicienne des Idées et des Nombres*, 1908; A. E. Taylor, *Forms and Numbers in Philosophical Studies*, 1934; also G. Milhand, *Philosophes-Géomètres de la Grece*, 1900. For (3) a systematic account of stereometry (going beyond the limits of this paper) is necessary.

<sup>34</sup> Aristotle *Metaphysics I* 990a30 and 991b26-30.

with the "Boundless."<sup>35</sup> Although they are really ratios, Plato calls them "Ideal numbers" because they are the "ideas" which generate the natural numbers. They do this not because there is an ideal number four to generate the natural number four, but because the atomic triangles and their interrelations, which the atomic ratios define, "cut" the "indeterminate dyad" in a peculiar way to give rise to the unnatural sequence of the generation of the natural numbers, to which Aristotle later objects.<sup>36</sup>

It is to be noted that the atomic ratios and their corresponding atomic triangles put an end to division. The intuitively-given undifferentiated continuum is, as with Democritus, divisible without limit. Hence Plato calls it the "indeterminate dyad." Since it also permits the one ideal ratio to define similar triangles of large or small sizes, he also termed it "the dyad of the great and small."<sup>37</sup> Because the elementary ratios are internally related to each other in the regular solids and in the structure of nature as a whole, Plato referred to the system of ratios as "the One." The compounding of the elementary ratios gives complex "ideas." In his famous lecture "Concerning the Good" in which, for the first time, he

<sup>35</sup> See Q. u. S. I. p. 19.

<sup>36</sup> Aristotle, *Metaphysics* M. 1081a23 and b16; 1081b2,30.

<sup>37</sup> I find myself unable to subscribe to A. E. Taylor's ingenious theory that the "dyad of the great and the small" is to be associated with the modern notion of two converging series of rational numbers which define an irrational number. This would mean that the indeterminate dyad was defined in terms of the natural numbers since they are necessary to define the rational numbers which define the series. But this was not Plato's method. Instead of using the natural numbers to define the ideal numbers, some of which Taylor identifies with modern irrationals, Plato used the "ideal numbers" and the "dyad of the great and the small" to generate the natural numbers. Moreover, Taylor's theory, if I understand it correctly, would give the "dyad of the great and the small" at least a small amount of form. In the lecture on the good, Plato tells us, however, that it is a material principle and that the One is the source of form. We shall attempt to indicate later in this paper why Plato conceived of the "indeterminate dyad" as a generating principle, and why Plato's theory does have certain partial similarities with the modern theory notwithstanding the fundamental differences between them. For further discussion of Taylor's theory see O. Toeplitz, *Q.u.S.I.*, and the previously cited monograph by Hasse and Scholtz.

stated the fundamental principles of his philosophy, Plato said: "The ground principles (*ἀρχαί*) of all things, and also of the ideas, are the One (*τό ἓν*) and the indeterminate dyad (*ἀόριστος δυάς*) or the great and the small."<sup>38</sup> Not only all perceived things but also all ideas reduced to the system of ratios<sup>39</sup> and the intuitively-given continuum.

It is important to note that the "ideas" derive solely from the One. Plato made this clear in the same lecture when he added that the dyad of the great and the small provides the matter, and that the One is the source of concept and form.<sup>40</sup>

But before Plato could regard the atomic ratios as internally related in a single proportion or *λόγος*, more than a knowledge of the foundations of mathematics and of stereometry was required. It was necessary to show that the whole of the universe, as well as its molecular constituents, involved irreducible geometrical form. This is the crucial issue between Plato and Democritus. The Democritean atomic theory also required that all things in the universe should be definable in terms of ratios. The arithmetical physical atoms in aggregating would combine in different numbers of units and thereby give rise to ratios between things. But for Democritus the ratios were secondary to the numbers,<sup>41</sup> since the atomic units would remain, while the ratios came and passed with the shift in combination, produced by the motion of the atoms. Plato puts the issue clearly in the *Philebos* 28d. After giving a general account of the "mixing" of the atomic ratios with the "Boundless" which we have described in greater detail in the previous paragraph, Plato says: "Let us begin by asking whether all this which they call the universe is left to the guidance of an irrational and random chance, or, on the con-

<sup>38</sup> See Toeplitz's excellent marshalling of the sources in *Q. u. S. I.* p. 19.

<sup>39</sup> The reader may wonder, if this is the case, why tradition came to refer to (*τό ἓν*), not as a collection of *λόγοι* but as a single *λόγος*. A systematic study of Greek stereometry, which we hope to develop in a later paper, should answer this question. There is a sense in which the atomic ratios are defined or determined by a single ratio.

<sup>40</sup> See *Q. u. S. I.* pp. 19-20.

<sup>41</sup> See E. Frank, p. 158.

trary, . . . ordered and governed by a marvelous intelligence or wisdom." In other words, is the relatedness of the ratios a derived notion or an irreducible primary concept?

Greek astronomy answered this question in favor of the latter alternative. In bringing its empirical observations into order to produce the first empirically verified, mathematical astronomical theory, this science was led to the conception of the entire universe as a single system of irreducible perfect geometrical spheres related to each other as to their distance by irreducible ratios. Not merely the part but the whole was based upon an irreducible *λόγος*. This is one reason why Plato referred to the relatedness of the atomic ratios as "the one," and why he placed astronomy near the top of the dialectical ladder as the last of the empirical sciences which one studies before applying "dialectic," or formal logic, to the scientific concepts to produce the consistent systematic intellectual outlook which constitutes the Idea of the Good.

It is important to note that for Plato the word "good" has no meaning by itself. There are, for man before he has climbed the dialectical ladder, no independently and intuitively-given absolute, known values. There are no ethical ideas existing independently of the fundamental concepts of the natural science. The idea of the good is not an independent idea within one's philosophy, since for Plato all the constituent "ideas" of one's philosophy are mathematical ratios. The idea of the good is all these mathematical "ideas" considered in their irreducible order and unity as "the one." Plato said this explicitly in his famous lecture "On the Good." The reports on this lecture examined recently by Toeplitz,<sup>42</sup> agree upon the following account: Many attended the lecture "under the impression that they would obtain some of the human values, such as riches, health, power, or above all a wonderful blissfulness. But when the exposition began with mathematics, numbers, geometry, astronomy and the thesis (*τὸ πέρασ ἀγαθὸν ἐστὶν ἕν*) 'The class of the limit conceived

<sup>42</sup> Q.U.S. I. 18-27.

as a unity is the good'<sup>43</sup> the surprise became general. A part lost interest in the subject, the others criticized him."<sup>44</sup> It is to be noted that "the limit" and "the one" are identical.

The crucial role which astronomy played in establishing this ethical and metaphysical and mathematical principle of unity is stated most clearly in the *Epinomis* 991-992a. There, after specifying the foundations of arithmetic as "the most important and first" subject to study if man is to "become a single, instead of a multiple, personality" and "be happy and blessed," and after adding stereometry, Plato, or someone very close to his thought, prescribes astronomy, "for then will be revealed a single bond of nature binding all these together." Nothing ever occurred in Greek astronomy to cause science to differ with Plato upon this point. Even Copernicus and Kepler took this conception for granted.

It is this single changeless primary "bond" of unity, unequivocally confirmed by empirically verified astronomical theory, which finally justified Plato's contention that a world *λόγος*, one with, and, as the doctrine of reminiscence expressed it, potential in, the human soul, is the ultimate and irreducible source of the determinateness of all things.

To understand the meaning of Plato's ethical, epistemological and metaphysical theory and of Aristotle's physical and metaphysical doctrine of the Unmoved Mover, and to appreciate why the rational principle of unity, explicit in them both, moved with such authority during the Middle Ages and the scholastic period, is to realize that Greek mathematics made ratio (*λόγος*) rather than number primary and that Greek astronomy revealed a single irreducible *λόγος* for the universe as a whole.

Our examination of the scientific basis of Plato's philosophy must not come to an end without a consideration of the epistemological problem. No philosopher has ever understood this problem in all its ramifications more thoroughly

<sup>43</sup> I follow Toeplitz' translation here. See Q.u.S. I. 23.

<sup>44</sup> Q.u.S. I. 19.

than did Plato. His technical mathematical knowledge together with his philosophical capacity to think things through, taught him that the metaphysical problem of the one and the many is at the heart of the matter.

Faced in the world of daily life with the obvious presence of many different people with their diverse and conflicting opinions, and coming after Democritus with his pluralism and attendant nominalism, Plato grasped the full force and significance of the Sophists' contention that "man is the measure of things." He saw that if reality is nothing but a many, their contention is unanswerable. No appeal to one's practical experience, or to one's inner consciousness, or to the logical presuppositions of one's thought will release one from the complete skepticism in knowledge, the relativity in ethics, and the chaos in politics which unqualified pluralism involves.

The reason is clear: Unrestricted pluralism leaves the knowing subject so completely other than anything it pretends to know, that there is no basis for epistemic connection with any possible object of knowledge except itself, and, hence, no justification for the supposition that one's ideas hold for anyone else, or correspond to the formal character of anything beyond the knower. If this be true, the Sophists are right: Man is the measure of things, the opinion of one person, on any subject, is as good as that of another, and no ideas, holding for all, exist to provide the agreement necessary to constitute government.

The conclusion is obvious: If knowledge is possible and there is to be some measure for men, necessary to distinguish social good from social evil, and provide a pattern for the state and a criterion for justice, the knower must be, in some ultimate metaphysical sense, in part at least, one with what he knows; the universe, binding man within it, must be in some irreducible sense a unity,<sup>45</sup> rather than a thoroughgoing plurality. Only in this way can man be left in nature so

<sup>45</sup> Compare A. N. Whitehead. *Process and Reality*, 1929. Ch. II. In particular p. 112.

that his knowledge can be objectively valid, and socially authoritative. Only upon this monistic basis can there be "*the* idea of the good" instead of many conflicting purely relative opinions of good.

Once this essential connection between ethics and politics and a solution of the epistemological problem in terms of at least a partial monistic answer to the metaphysical problem of the one and the many is recognized, the point in Plato's insistence, in the Republic, upon the education of moral philosophers and statesmen in the natural sciences takes on new meaning.

An appeal to one's introspective experience will reveal nothing but the emotional embodiment of the relative opinions of the moment. Daily life and social phenomena, likewise, embody the many opinions of the many individuals. We see many different people. We note their many conflicting opinions. The social institutions and movements in which they participate embody and exemplify these conflicting opinions. All this suggests to us that the pluralistic principle is primary. Hence, there is no hope of gaining ethical or political theory which will command common agreement and consent by appealing to introspective psychology or to an independent empirical science of government or sociology. All such empiricism would give, are the shadows on the wall of the cave in which circumstance has chained us. Hence, the only hope of revealing the irreducible principle of unity necessary to validate communal knowledge and sustain social ethics and government, is to tear people away from the "chains" which affectionately attach them to the purely personal opinions which subjective feeling and the circumstances of social experience engender, by directing their attention to the single *λόγος* of the universe, which a study of Greek mathematics and astronomy reveals. Professor Ferguson's recent study<sup>46</sup> of the text of the Republic has shown that this

<sup>46</sup> A. S. Ferguson. Plato's Simile of Light. The Classical Quarterly XV 131-152 and XVI 15-28.

is Plato's meaning. The key to the epistemological, as well as the ethical and political problem is to be found "only" in the natural sciences.

The central doctrine in Plato's theory of knowledge is the transcendental theory of ideas. The doctrine of reminiscence is a necessary consequence of this theory. Ideas were transcendental for Plato because they were *discovered* "only"<sup>47</sup> in nature, and because they were the irreducible mathematical "ratios" which constitute the determinate character of all natural objects, including man. The doctrine of reminiscence was valid because the same small finite number of elementary ratios which constitute everything also constitute man.<sup>48</sup> Hence, the fundamental "ideas" which man discovers by studying the foundation of arithmetic and stereometry were in him as potential objects of knowledge from the beginning. Also the elementary "ratios" are internally related by the irreducible single structure of the universe as a whole, which astronomy reveals and a systematic study of stereometry defines. Hence the particular local complex of atomic "ratios" which defines the determinate nature of the individual man is internally related to the other complex stereometric forms of the rest of the universe. In fact, there is no meaning to the existence of man apart from this wider *λόγος* of the whole universe. The local character of an individual man is but one factor in his nature. Hence, only by knowing the mathematical structure of the whole universe does man come to a full knowledge of himself. When this happens, skepticism loses at least some of the plausibility it had at the bottom of the dialectical ladder.

<sup>47</sup> Republic VII 533.

<sup>48</sup> Timaeos 69, 73, 80, 82. The distinction between man's body and soul, to which these passages also refer, centers in part in the distinction between the ideal *λόγος* of which there is but a single instance and the local complex of atomic triangles ("mathematicals") in "the class of the mixed" of which there are many instances. (See Aristotle Metaphysics I 991b 17-20.) It also centers in the distinction wholly within "the class of the limit" between the formal one and the formal many. A study of the systematic relationship between the regular solids is necessary to make this clear.

The knower is no longer so completely other than any possible object of knowledge that any basis for epistemic connection is lacking.

Furthermore, when this conception of the self in its relation to the universe arises and becomes established as a scientific truth which is independent of the variations in human opinion, it possesses the authority necessary to command common agreement. When this occurs it tends to be taken for granted; what one thinks and does tends to proceed from this assumption. This is especially true if the belief is accepted generally. In other words, it hovers over the background of one's thought and action like the sun which illuminates and thereby determines what one sees and does. (Republic VI 508, VII 517.) Any conduct out of harmony with it seems incorrect, or in other words "bad"; any conduct in accord with it seems correct, or, as one says, "good." This is the reason why Plato called the *λόγος* which is the mathematical structure of the universe, "the idea of the good." There was nothing especially blissful about it; it was the mathematical bond of all the ratios. But because it defined a correct philosophical conception of the nature of things for the science of the time, and, hence, could serve as a reasonably authoritative assumption from which to proceed in one's personal and social conduct, it gave the criterion for the only meaningful distinction there is between *the* good and *the* bad, which must hold for everybody. As we have indicated previously, the idea of the good, for Plato, is not an independent idea along with others in one's philosophy; it is one's philosophy. This is the reason why philosophy is absolutely essential for sound ethics or good government.

It is to be emphasized, therefore, that the doctrine of reminiscence does not mean that one can find "the idea of the good" in the introspective experience of the individual before he has studied science. The ratios which define the atomic triangles and the irreducible mathematical unity which relates them to constitute the specific order of the universe is not

given so cheaply, to even the most pious and well-intentioned saint. Only at the top of the dialectical ladder when the scientific conception of the mathematical nature of things has been so thoroughly accepted and understood that in the process of acquiring it one has incorporated it in one's very being so that one acts upon the basis of it unconsciously, and constitutes one's inner life in terms of it, — only then, does "the idea of the good" become a frenzy as well as a theory, so that one's instincts and one's subjective intuitions can be trusted.

To suppose that one can come to the good introspectively through oneself is to seek for "ideas" in the world of sensation where "genuine" knowledge does not exist, and to expect to find the good at the bottom of the dialectical ladder. It is also to confuse vague symbols referring to irrational items in the sensuous continuum, with the one "idea," which mathematics alone can reveal, and which the mathematical intellect alone can understand. Moreover, if by a miracle one should find the idea of the good in this way, one would destroy rather than save morality and the state by this method, for the appeal to personal subjective experience would justify Thrasymachos' opinion as well as Plato's, and thereby give rise to a pluralism of opinions of the good measured by men, instead of the one "idea of the good" which measures men. This is another reason why ethics must be reared upon a naturalistically-grounded philosophy which permits of a criterion for truth that is independent of the opinions of men, and why philosophers and statesmen must be taught the subjects prescribed by Plato in the Seventh Book of the Republic.

In this connection, Professor Ferguson's article, to which we have previously referred, should be studied with great care. It shows that the portion of the determinate intuitive continuum which is given introspectively has a fundamentally different status in knowledge from the portion which exhibits nature. The order of the former portion of the sensuous

continuum is untrustworthy as a basis for testing hypotheses proposed by the scientific intellect, since it is determined by human opinion; only the naturalistic portion can be trusted since the order of nature exists quite independently of any opinions or hypotheses of men. This is the reason why knowledge does not begin until one is released from one's "chains" and taken out of the cave.<sup>49</sup> To go out of the cave is to direct one's attention away from the subjective toward the objective portion of the differentiated world of sensation. The climb up the dialectical ladder does not begin until this has been done; it consists in observing the continuity and the diversity within the sense world of nature, in discovering incommensurable magnitudes and thereby coming to the distinction between the intuitively-given undifferentiated continuum and the purely conceptually-given mathematical "ideas" which give the continuum its determinateness, and in analyzing the stereometrical forms of the possible three-dimensional objects to find the fundamental mathematical concepts and their relatedness in "the one," in terms of which the "appearances" in nature "can be saved."

Plato sees that it is futile to hope to escape skepticism by appealing to man's so-called moral experience and suggesting that a transcendental idea of the good is presupposed. This leaves Thrasymachos on as secure a footing as Socrates. It is necessary, if the Sophists are to be answered, to point out some specific precise "idea" and to show that it is demanded by the distribution of the observed appearances in nature and is a primary scientific concept, holding for all men and applying to the whole of the universe. The *λόγος* discovered by Greek mathematics and astronomy provided this "idea."

This is the reason, never made clear by the modern moral philosophers, who have written so much on the subject, why Plato taught men "only" <sup>50</sup> these sciences brought into "intercommunion and connection" by dialectic in order to make

<sup>49</sup> Republic VII 514, 515 ff.

<sup>50</sup> Republic 533a 8-10. Also Aristotle Metaphysics I 992a 33.

them good. The "idea" of the "good" is in all men from the beginning, but it is so obscured by the opinions and social circumstances which constitute the determinateness of the center of one's ordinary consciousness that only a study of the natural sciences will bring it out. As Plato says in the *Timaeos* god: A man can know the good only if he be converted, and he can be converted only "by correcting those corrupted courses of the head" and "assimilating the perceiver to things perceived by learning the harmonies and revolutions of the whole." Only if this be kept in mind will one understand why Plato could consistently hold the doctrine of reminiscence and at the same time assert that one can come to the idea of the good "only through the study"<sup>51</sup> of arithmetic, stereometry and astronomy.

Our examination of the specific content of these sciences has revealed the scientific basis of Plato's treatment of the epistemological and the ethical problem. It has also disclosed the two factors which constitute the universe and everything within it, including not only man but also the mechanics of the processes of knowledge. These two factors are the intuitively-given undifferentiated continuum of the world of sensation and the purely conceptually-given mathematical One. The former factor was distinguished from the observed differentiations which it contains, by Democritus in order to reconcile the existence of incommensurable magnitudes with the atoms known only by the scientific intellect. The mathematical *λόγος* was introduced by Plato as a modification in the Democritean theory of "indivisibles" in order to bring incommensurable geometrical figures within scientific theory, and in order to account for the irreducible mathematical structure of the universe required by astronomy. Plato called the observed continuum the "dyad of the great and the small" or "the indeterminate dyad." In the "Philebos," it appears as "the class of the infinite." In the *Phaedros* it is the frenzy principle. In the *Timaeos* it is the Receptacle.

<sup>51</sup> Republic 533a 8-10.

He called the mathematical *λόγος*, "the one," and in his famous lecture "On the Good," said it was "the idea of the good." In the *Philebos* it appears as "the class of the limit," and the pure wisdom. It is important to note that both factors provide a principle of unity. The intuitively-given continuum is a unity,<sup>52</sup> the One is a unit. We shall henceforth refer to them as the sensuous one and the formal one.

However, it is essential to recognize that another unity is involved. Plato tells us in his lecture on the good that the sensuous one and the formal one are "the ground principles of all things." This means that everything is constituted of them without exception, including the soul of God and of man. This must be the case since reality is constituted of nothing but the sensuous one, and the formal one which "mixes" with it to differentiate it into the specific natural objects which we observe in nature. This actual differentiated continuum of nature with all the objects in it which we observe is what Plato calls "the class of the mixed."<sup>53</sup> Put more concretely, this means that when the tetrahedron, defined by a certain proportion of the elementary ratios or "ideal numbers," "mixes" with the undifferentiated intuitively-given continuum, perceived fire appears as a differentiation of the continuum; when the entire *λόγος* mixes, all the observed diversity and complexity of perceived nature appear. In this process, which Plato terms "a generation into essence,"<sup>54</sup> the formal one contributes the form, the sensuous one contributes the matter.

But this is not the only function of the "Boundless." It is a sensuous one as well as an indeterminate one. This sensuous purely intuitively-given character of the indeterminate "Boundless" insures that it is the source of *immediate* experience, or, in other words, consciousness. Man is conscious not because he is an independent mental substance, but be-

<sup>52</sup> Compare A. N. Whitehead. *Adventures of Ideas*. pp. 171, 192.

<sup>53</sup> *Philebos* 26, 27.

<sup>54</sup> *Philebos* 26d 8.

cause he, like everything else in the universe, is constituted of the "Boundless."<sup>55</sup> This is why Plato called the sensuous one a ground principle, instead of regarding it as a mere sum of arithmetical units as did the early Pythagoreans, or treating it as a purely phenomenal addition, projected as an excretion from metaphysically independent souls, as so many moderns have accused him of doing. The lecture "On the Good" clearly indicates that the soul is not a third ground principle in addition to the "indeterminate dyad" and "the one." There are only two ground principles of all things.

An appreciation of this fact brings us to the third aspect of unity in Plato's theory. There are only two ground principles. But they do not exist separately. Not only is each one a unity, but there is a unity between them. The sensuous one and the formal one are internally related. Hence they *must* mix to generate determinate experience. This internal relatedness of the two ground principles is what constitutes the soul and makes mind an irreducible fact in the universe. It is this internal relatedness of the two sensuous and formal materials, out of which everything is constituted, which causes Plato, in the *Philebos*, to speak of mind as the cause of their mixture.

In this mixture, not merely the local complex of stereometrical, regular solids which define the determinate character of the individual man, but also the *λόγος* of the whole universe, combines with the sensuous unlimited continuum. Hence there will be not merely the many human souls, but also the Divine soul.<sup>56</sup> It is important to note that both the sensuous

<sup>55</sup> For contemporary suggestions or examples of this conception see A. N. Whitehead, *Process and Reality*, pp. 112, 124-6, 472; and the definition of the psychical in the author's *Science and First Principles*, 1931, Ch. VI, especially p. 256. Also Charles Hartshorne, *The Philosophy and Psychology of Sensation*, 1934; and B. W. Brotherston, *Empiricism and Unity*, *Journal of Philosophy* XXX 141-9.

<sup>56</sup> For an excellent and scholarly account of these more purely psychological and theological aspects of Plato's philosophy see R. L. Calhoun's *Plato as Religious Realist* in *Religious Realism* by D. C. Macintosh and others, 1931, pp. 195-251; also R. Demos, *Journal of Philosophy* XXXI 337-345. In this connection it is to be remembered that none of these

one and the formal one are required to arrive at this conclusion. The formal one must exist if there is to be a determinate *rational* being in the whole of nature as well as in the part. The sensuous one must exist likewise in continuous extension throughout the universe to insure that nature as a whole possesses the sensuousness and the "frenzy" requisite to experience the determinate being that it is. Were the source of consciousness restricted solely to man, there would be human souls but no divine soul. It is the localization of the source of intuitive feeling and consciousness in the undifferentiated continuum of nature as a whole, which enables Plato, upon finding the astronomical *λόγος*, to assert that there is a world soul and a God. If the sensuous one existed without the formal one, there would be world experience but no determinate world; if the formal one existed without the sensuous one, there would be world structure but no world soul. Thus the existence of God as a frenzied soul rather than a mere mathematical ratio depends on the fact that the sensuous factor in reality is an unlimited continuum and a ground principle. It was this conception also, as E. Frank has shown, which kept sounds and music in nature, and made the notion of the music of the heavens quite natural to the Greek mind.<sup>57</sup>

Just as one must distinguish between soul and the rational principle in the soul, so one must distinguish between good

conceptions referring to factors with frenzy in them can be taken for anything more than mere opinion (hence, the *Timaeos* which treats of these matters deals only with probabilities) until the mathematical one in the class of the limit introduces the distinctions which the conceptions demand. Frenzy, or the "Boundless," introduces the infinite and "the more and less." Hence things in the class of the mixed suggest a countless number of hypotheses. It is because the *λόγος* in the class of the limit is absolutely determinate and because it defines *one* and only one theory, and because it gives the mixed world its determinateness, that it provides a criterion for determining the truth or falsity of statements referring to perceived objects or subjective intuitions in the class of the mixed. It is for this reason also that it is called the *measure* and is identified with the idea of the good. When we realize that for the Greeks mathematics gave but one formal system, Plato's confidence in it as criterion for truth, which must hold for everybody, becomes the more understandable.

<sup>57</sup> See E. Frank, pp. 1-19.

and the idea of the good. Soul is the formal one so internally and eternally related to the sensuous one that the two must mix to produce the differentiated continuum of immediate awareness as "a generation into essence." The soul is the sensuous one in immediate generative synthesis with the formal one. The rational principle in the soul is merely the formal one. Likewise, the idea of the good is nothing but the formal one. It is the one mathematical proportion<sup>58</sup> of the elemental ratios of the universe. It can be only this; otherwise, it would not be an "idea." The good, on the other hand, is the "idea of the good" merged with frenzy. In other words, it is the divine purely formal *λόγος*, merged with the unlimited "boundless" to generate the Divine frenzy. As the language of the Philebos would express it, the idea of the good is in "the class of the limit," and the good is in "the class of the mixed."

Only "ideas" can be conveyed and transmitted to others. Anything in the class of the mixed, whether it be observed nature or the human soul or the Divine frenzy, has to be experienced immediately to be known. One can tell no one who has not had the experience what blur is or what an observed, as opposed to a conceived, mountain is, or what good is, or what the soul is, or what God, or the Divine frenzy, is. Such matters, as well as the story of their creation, have to be conveyed in myths. There can never be identity, there must always be "distance,"<sup>59</sup> between the object and its symbol. The reason for this is that all these factors have frenzy in them; they contain the sensuous one as well as the effects upon it resulting from its mixture with the formal one. This presence of the "Boundless" in all immediately sensed factors gives them an ineffable character, which no ideas can express

<sup>58</sup> Strictly speaking the One is a single ratio, as the word *λόγος* indicates. The atomic ratios which define the atomic triangles do not merely combine to produce a *λόγος* for the universe. Instead there is a sense in which a single ratio in the One defines the atomic ratios. A systematic outline of Greek stereometry is necessary to clarify this point.

<sup>59</sup> Scott Buchanan, *Symbolic Distance*, 1932.

or convey. Symbols can be assigned to these factors to call them to mind after they have once been experienced but no ideas can express them. They cannot be known by the intellect.

On the other hand the idea of the good, the idea of God, and the idea of the soul, can be expressed; — being constituted of the elementary ideas or ratios, they can be expressed in terms of them. Since there is no ineffable contribution of the "Boundless" in them, they can be expressed in terms of the elements of all form which mathematics reveals. But even this would not be possible did we not have the *λόγος* in us potentially before we begin our study of the sciences. After all, a blind man cannot see. Hence, it is the formal ground principle that constitutes and measures men, and not man, in the solipsistic isolation of the Sophists' metaphysical pluralism, who constitutes all theory and measures all things.

In the unity of the sensuous continuum, and the unity of the cosmic mathematical ratio, and the unity of these two "ground principles," Plato found the evidence necessary to give the metaphysical problem of the one and the many a monistic answer. Need we now wonder that he regarded arithmetic, stereometry and astronomy, supplemented with the formal dialectic, which expresses the elemental concepts of these sciences in a consistent theory, and carries that theory to its epistemological, ethical, political and religious consequences, as a better foundation for ethics, politics, epistemology and religion than the deliverances of introspection or the confusions and conflicts of the statesman's chamber and the market place?

The more detailed epistemological consequences of this mathematical philosophy are suggested in the *Theaetetus*. The first point to note is that the division between the conscious and the non-conscious, centers in the distinction between the sensuous one and the formal one, rather than in the distinction between the knower and the object of knowledge. Hence, the knower and the object known, or, as

Plato terms them, the patient and the agent, are on precisely the same footing. The knower is never left in metaphysical isolation in a conscious world so private and purely personal that one must wonder how knowledge of anything but himself or his impressions ever gets into it. Since the distinction between the sensuous and the non-sensuous centers completely in the difference between the two "ground principles of *all* things," man and any other thing in nature have precisely the same status. The idea, or more specifically, the complex of elementary "ideal numbers" constituting the stereometrical solids which define man as he is known with the intellect, mixes with the "Boundless" to produce man as he experiences himself immediately in the world of sensation. Perception is not a purely local faculty of a metaphysically independent local soul; it is a mere expression and inevitable consequence of the internal relatedness of the sensuous one and the formal one. In the same way, the complex of ideal numbers defining the stereometrical solids which constitute this sheet of paper, as it is conceived by the scientific intellect, mixes with the "Boundless" to "generate" the white sheet of paper which we see before us in the world of sensation. But these two events do not occur in isolation. The complex of ideal numbers which defines the knower as "idea," is internally related to the complex of ideal numbers which defines the sheet of paper as "idea," and the entire complex merges with the "Boundless" in one event. When this happens the "patient becomes percipient" and the paper becomes white. (Theaetetus 182.)

This should clarify Plato's theory of ideas. The idea of man is not the general term which designates all perceived men. The idea of myself is not what I sense when I consider myself introspectively. The idea of fire is not the general term (or any notion in the mind of which the general term is a symbol) referring to all cases of perceived fire. Nor is the idea of fire that notion given by a definition which distinguishes fire from all other factors in the universe, in terms of

perceived properties. All the so-called concepts which we use in ordinary discourse, and which most modern philosophers have in mind when they refer to Plato's theory of ideas, — all such notions are not ideas, in the Platonic meaning of the term. They are merely nominalistic terms referring to factors in "the class of the mixed." Ideas are purely in the class of the limit and they cannot be illustrated by pointing to anything immediately sensed. The "mathematical" of fire is the tetrahedron, and the "idea" of fire is the proportion of the ideal numbers which defines the tetrahedron. The general nominalistic term which denotes the perceived fire is a "sensible" rather than either a "mathematical" or an "idea." Terms referring to "sensibles," such as perceived fire, or observed men, or my intuitively-given self, are the words of ordinary discourse. Graphical symbols referring to "mathematics" are the geometrical figures one draws on the blackboard. And "ideas," whether they be the idea of the fire, the idea of myself, or the idea of the good, are ratios which only an analytical mathematical symbolism can express and only the pure scientific intellect can grasp. Is it any wonder that Aristotle in speaking of Plato said that "Mathematics has been turned by our present day thinkers into the whole of philosophy?"<sup>60</sup>

It is because only the mathematical ratios and the "Boundless" are primary in their own right, and because the sensed things exist only in the perception in which they occur as a result of the "generation into essence" resulting from the mixture of the λόγος and the "Boundless," that the differentiated sense world which contains introspectively-given selves and perceived white sheets of paper and perceived fire, is in any sense a phenomenal world. It is phenomenal, not because it is the private projection of a local knowing subject, but because it is the result of the mixture of two universal ground principles, instead of being a ground principle itself.<sup>61</sup>

<sup>60</sup> Metaphysics I 992a 30.

<sup>61</sup> This is the basis of Aristotle's criticism of Plato in Metaphysics 988b.<sup>61</sup>

This is the basis of Plato's criticism of art. He was a great artist himself. He had a thorough appreciation of literary form and dramatic effect. But he saw that it is of the very essence of art that it works with immediately experienced materials or with the symbols of ordinary discourse which refer solely to such immediately given factors. This is very good when one uses it as art, merely for what it is, but when one uses it for education as if it gave "ideas," then it is, to say the very least, misleading.

The key to the entire conception is in the theory of the scientific concept introduced by Democritus in order to reconcile the discovery of incommensurables with the requirements of arithmetic. According to this theory, science states its laws and principles in terms of concepts referring to unperceived elements and not in terms of the immediately observed differentiated continuum. The differentiations in the continuum must be the criterion for the truth of any scientific theory, but the theory itself is not stated in terms of notions which either the continuum or its observed differentiations reveal immediately. The observed world *suggests* the concepts of scientific theory, but it does not *contain* them. We do not see atoms, we do not see atomic triangles, we do not observe ratios, yet for either Democritus or Plato these are the elements in terms of which scientific theory is expressed. Notwithstanding this difference between the basic elements, or "ideas," to which scientific theory refers, and the perceived factors of the observed world, these scientific theories can be put to an empirical test. This is done by correlating the entities of the theory known by the intellect with specific differentiations in the observed world which are known "by sensation." For example, in Plato's theory, the tetrahedron is associated with perceived fire, the cube with perceived earth, and so on. The theory is empirically verified if, at every place where it prescribes the presence of a tetrahedron, perceived fire is observed; it is condemned if the perceived differentiations in the observed continuum of nature do

not occur in accordance with the corresponding theoretical distinctions which the theory prescribes. At this point Plato foresaw the method of modern mathematical physics.<sup>62</sup>

It has been necessary to treat Plato's theory of ideas in so much detail and to keep Plato's philosophy in such intimate association with Democritos' particular solution of the problem raised in Greek mathematics by the discovery of incommensurables, because otherwise we shall not understand Aristotle. Unless we comprehend precisely what Plato meant by an "idea," as opposed to a "mathematical" or a "sensible," and appreciate how his conception arose in order to remove the artificiality in the Democritean theory of incommensurables, while still retaining the Democritean distinction between "scientific objects" and "sense objects,"<sup>63</sup> and still affirming the Democritean doctrine that there are elemental magnitudes, we shall never get the point of Aristotle's criticism of Plato, or realize that when Aristotle talks about an idea, he means something entirely different from what Plato meant.

A complete account of Plato's philosophy should designate the mathematical meaning of his doctrine of measure, and the more specifically stereometrical meaning of his idea of the good. This would indicate the specific distinction within the formal principle between what may be termed the formal one and the formal many. Without this, certain statements which he made in the *Philebos* concerning the class of the limit, and the myth of creation in the *Timaeos*, cannot be comprehended; also, an adequate account of the "generation of the odd and the even" natural numbers cannot be given. But such an exposition would require the inclusion of a systematic outline of the science of stereometry and take us beyond the limits of this paper.

One would like to conclude one's remarks on Plato with

<sup>62</sup> See A. N. Whitehead, *Adventures of Ideas*, 194-6.

<sup>63</sup> Compare A. N. Whitehead, *The Concept of Nature*, 1920, Ch. VII.

the statement that his theory solved the fundamental problem in the foundations of Greek mathematics, which it was introduced to resolve. Certainly such a scientifically technical and philosophically comprehensive theory deserved this triumph. Nevertheless, this was not its fate. Although the general outlines of Plato's theory were to persist because of its emphasis on the general concept of ratio and its astronomical verifications, it failed as a theory of the fundamental concepts of mathematics. The story of this failure is the story of the origin of Aristotle's philosophy.

The tale is a very brief one. Between the thought of Plato and the thought of Aristotle stands Eudoxos. The final solution of the problem which Democritos and Plato attacked came with the Eudoxian general theory of proportions. This theory, together with the method of exhaustion, entails the Anaxagorean principle of divisibility without limit.<sup>64</sup> The Democritean theory of indivisible extended atoms and the Platonic theory of elemental extended atomic triangles with their atomic ratios, is incompatible with this Anaxagorean principle. Even though Aristotle had been trained in the Platonic theory for twenty years, he could not escape the obvious conclusion: Democritos and Plato "are forced to affirm that which mathematics has denied."<sup>65</sup>

It is this unequivocal statement made again and again by Aristotle in his physical treatises, and considered by him to be so obvious to his contemporaries as to need no argument, which prevents us from going all the way with Toeplitz in his thesis that Plato's philosophy incorporated the Eudoxian general theory of proportions,<sup>66</sup> or with A. E. Taylor in his contention that Plato attained the modern theory of the irrational number.<sup>67</sup> Were either contention correct Plato would have had to admit divisibility without limit and reject his atomic triangles. That Plato did not do this is precisely

<sup>64</sup> Euclid, X 1.

<sup>65</sup> Aristotle De Gen. et Cor. 325b25; De Caelo 303a20 and 306a27.

<sup>66</sup> Q.u.S. I.

<sup>67</sup> Forms and Numbers in *Philosophical Studies*, 1934.

the point of Aristotle's objection to Plato's mathematical theory.

There can be no doubt, however, as Taylor and Toeplitz have indicated, and as the *Epinomis* source 990d, or as examination of the atomic triangles of the *Timaeos*, shows conclusively, that Plato's theory did incorporate incommensurable as well as commensurable magnitudes. We are required, therefore, to regard Plato's mathematics as an intermediary stage on the way from Democritus to Eudoxos.

This is not to make Plato's contribution insignificant. In fact, it may well be that he provided one of the two fundamental ideas necessary to define the triumphant Eudoxian theory,—namely, the notion of ratio as more fundamental than arithmetical number, and as applicable to incommensurable as well as commensurable magnitudes. The ratios which define the atomic triangles possess such a character. But this is not sufficient to give the Eudoxian theory of proportions. In addition there must be the definition of "sameness of ratio" which appears in the fifth definition of the Fifth Book of Euclid's *Elements*. Tradition accredited this to Eudoxos.<sup>68</sup>

This Eudoxian definition is exceedingly important for logic. The key to the definition is in the words "any equimultiple whatever." The word "any" indicates that Eudoxos solved the problem involved in defining a mathematical operation which can employ the generalized concept of ratio, by introducing the notion of the variable.<sup>69</sup> This is the first time that the variable appears as an essential factor in the mathematician's definition of the fundamental concepts of this science. It appears again in modern times when Weierstrass gives the rigorous definition of the mathematical concept of limit,<sup>70</sup> which in turn, in conjunction with the need for a rigorous definition of continuity, forced Dedekind and Cantor to the definition of the real number and the modern theory

<sup>68</sup> See T. L. Heath, *A Manual of Greek Mathematics*, 1931, p. 190.

<sup>69</sup> See A. N. Whitehead. *Introduction to Mathematics*, 1911. Ch. II.

<sup>70</sup> *Ibid.* Ch. XV, especially pp. 226 and 234.

of the irrational.<sup>71</sup> It is this use of the variable by both Eudoxos and the modern mathematicians which makes it possible for those who come after Eudoxos and Weierstrass to view the Platonic notion of ratio as in certain respects formally equivalent to the modern theory of the irrational number.

However, there is evidence that the Greeks did not view it in this way. To the very end, as M. Cantor has emphasized,<sup>72</sup> the concept of number was restricted solely to the natural numbers. Even zero was not a number for Greek mathematicians. Moreover, there are differences as well as identities between the Eudoxian and the modern use of the variable. This appears when one considers its range. In order to attain the modern theory of number which enables one to arithmeticize geometry it is necessary to extend the range of the variable so that it can take on all the real numbers as its values. This wider generalization of the notion of the variable never occurred to the Greeks. In the crucial fifth definition in the Fifth Book of Euclid the range of the variable is restricted to whole numbers. It is the particular form that this restriction took on, which convinced Greek mathematicians that the concept of ratio holding between *magnitudes* is more fundamental than the notion of number. Only when the magnitudes are commensurable is the concept of number adequate. Instead of taking number as the fundamental concept and generalizing it, as did the Moderns, the Greeks brought commensurable and incommensurable magnitudes under a single mathematical theory by restricting the concept of number to natural numbers and generalizing the concept of ratio. This gave them the equivalents of our rationals and irrationals, but they were never regarded as numbers. Instead, they were thought of as ratios between two different types of intuited magnitude. What this means

<sup>71</sup> See B. Russell. *Principles of Mathematics*, 1903. Chs. XXIII, XXX, XXXIII, XXXIV, and XXXV.

<sup>72</sup> M. Cantor. *Vorlesungen über Gesch. der Math.* 1880. I. p. 159.

at bottom is that the Greeks never accepted G. Cantor's assumption of an actual infinite many.<sup>73</sup> The crucial significance of the fifth definition of the Fifth Book of Euclid for the Greek theory is that it defined *a relation of equality* between *ratios of magnitudes* which was so general that it permitted mathematicians to operate with the generalized concept of ratio.

This important definition had one other consequence. In conjunction with the Eudoxian method of exhaustion, it entailed the Anaxagorean principle of divisibility without limit,<sup>74</sup> and, thereby, necessitated the reconstruction in philosophy and the designation of the inadequacies in Plato's theory, which we find in the books of Aristotle.

Although Aristotle was undoubtedly led to his new philosophical theory by biological as well as mathematical considerations,<sup>75</sup> the requirements of the Eudoxian theory of proportion alone were sufficient to dictate the fundamental principles of his system. If the atomic triangles with their ideal ratios are not the fundamental constituents of nature then the Platonic identification of ideas with these ratios must be replaced by a different theory of ideas. Also, the Democritean and Platonic distinction between the sense world and the "intelligible" world is without basis. The only world which exists is the observed world. Hence, ideas must find their source and meaning in this world.

Furthermore, if there is infinite divisibility, nature must be potentially rather than actually divisible.<sup>76</sup> Otherwise, there would be an infinite number of parts in any finite magnitude and the contradictions indicated by Zeno would occur. This means that nature, considered solely under the category of quantity, is one rather than many. Hence, no

<sup>73</sup> See G. Cantor, *Gesammelte Abhandlungen*. 1932. pp. 370-376; and Aristotle *De Gen. et Cor.* 318a20-21.

<sup>74</sup> Definition 4 (of Book V) in *The Theory of Proportions* itself is often assumed to entail this. See Heath's *Manual*, p. 193.

<sup>75</sup> See F. S. C. Northrop, *Science and First Principles*, Ch. I, 17-23 and Ch. IV, 168-70.

<sup>76</sup> Aristotle *Physica* III, 207b and *De Gen. et Cor.* 318a20.

irreducible quantitative statements are valid. There is no real plurality beneath the irreducible continuum of nature to guarantee the validity of quantitative statements, apart from the perceived qualities which differentiate the continuum. Hence, actual quantity presupposes perceived qualities, and all quantitative propositions must be analyzable into elementary propositions which predicate a quality of a subject. It was knowledge, rather than ignorance, of the mathematics of his time which supported Aristotle in the formulation of his logic.

An adequate treatment of the bearing of mathematics upon Aristotle's philosophy would indicate how it leads to his theory of matter and form, and how the syllogism is an analogue in the actual world of Aristotelian ideas for the proportions in the *abstracted*<sup>77</sup> world of mathematics. It would indicate also how syllogistic connections between local Aristotelian ideas relate together into a wider single Aristotelian logos for the universe as a whole, much as Plato's atomic ratios were internally related in proportions to constitute the idea of the good. This unity of all Aristotelian ideas would be presupposed in all knowledge just as the Platonic "idea of the good" was a prerequisite for all science. This should clarify a troublesome point in Aristotle's *Metaphysics*. It has seemed that Aristotle criticized Plato's theory of ideas in the beginning of the *Metaphysics* only to return to the Platonic position in the end. At least some light is thrown on this difficulty, if we realize that the necessity of accepting the Eudoxian theory of proportions forced Aristotle to base his entire philosophy upon an entirely different theory of ideas than Plato's, which, nevertheless, was analogous to Plato's with reference to the status of irreducible form in the universe as a whole. However, an adequate account of the precise manner in which the de-

<sup>77</sup> In any discussion of the relation between Plato and Aristotle, it is essential to distinguish clearly between concepts derived from the sense world "by suggestion" and concepts given "by abstraction." In this connection see A. E. Taylor's *Platonism and Its Influence*. 1924. Ch. II.

tails of Aristotle's system developed out of the Eudoxian pre-suppositions must await another occasion.

It remains but to add that the Eudoxian theory of the foundations of mathematics was superseded in the modern period, when the rigorous formulation of the theory of the infinitesimal calculus led to the generalization of the concept of number to include rationals and irrationals, and to Georg Cantor's doctrine of the actual infinite. Recently, it has become evident that this theory also involves one in paradoxes. Hence, we are still confronted in science and philosophy with the same fundamental problem with which these thoughtful Greeks wrestled. This makes their contributions of present import, as well as of historical interest.

# THE ONE AND THE MANY IN PLATO

BY RAPHAEL DEMOS

ACCORDING to Plato, to be, is to be limited—that is, determinate. What does it mean to be determinate? To ask this question is to plunge into the center of Plato's metaphysics. Kant asked the question: what are the conditions of all experience? Plato's question is ontological: what are the formal conditions of being? Yet the epistemological problem is not excluded; for Plato, there is a parallelism between thought and being. Thus, the topic of this essay is an inquiry into the principles of determinateness, both in thought and in being.

We must point out that the topic of this essay is circumscribed. Anything is real by virtue of its participation not only in the Limit, but in the Good. Just now, we are concerned exclusively with the factor of the Limit, in other words, with the *formal* requisites of being (*Phileb.* 26 d).

Determinateness involves two things, division and integration. To be limited is to be both One and Many. There is the pole of analysis and the pole of synthesis. Division is the aspect of the world as consisting of distinct entities, each identical with itself and other than the rest. This is the phase of the Many, and is the basis of the intelligibility of things, for only in so far as we can distinguish things from each other, can we have clear ideas of them (*Rep.* 524 c). It is also the phase of Rest. Whatever is identical with itself, is itself simply, is a 'that,' and so, is at rest. Correlatively, we have the aspect of integration. The distinct things are in a reciprocal relationship; they participate in each other; they entail each other; together they form collections—*ἀθροίσματα*—classes, wholes. This is the phase

of the One in the real. It is also the aspect of power—*δύναμις*—and motion. In so far as entities commune with each other, they pass into each other, they act or are acted upon.

Plato states that the ability to organize things into a unity, and to divide them into many is divine (*Phaedr.* 266 b); and a person without this ability is like a man who is blind (*Phaedr.* 270 e).

Both the One and the Many are equally constitutive factors of being. The real is neither just a unity, nor simply a multiplicity. The One is a whole of parts; the Many are articulations in a unity. These two principles, or rather the single principle of the One and the Many applies to all realms of being. It constitutes the world of particulars; these enter into unities, that is, classes; conversely, the classes are divided into subclasses and finally into particulars (*Phileb.* 16 e). It constitutes the realm of forms; in virtue of the principle of the One and the Many, the forms are integrated under the Idea of the Good, and, conversely, the Idea of the Good breaks up into the many forms. It is the basis of the constitution of the soul and the state, whereby the soul is a unity of several parts that are bound together, and the state is an internal relationship of individuals, each of whom performs a specific function. It is what constitutes the cosmos into an organized whole, consisting of a variety of individuals, each of them reproducing the whole, and yet possessed of a distinctive nature. And in conclusion, it is a principle defining the total metaphysical situation, such that the ultimate nature of things consists of a plurality of factors, each relevant to the others and the whole.

In Plato, then, there are these two principles of reality: a thing exists in itself; a thing exists for other things. The first is the principle of non-being, the second is the principle of communion. (Thus, to know the nature of the soul, we must know the nature of the universe. *Phaedr.* 270 c; *Phileb.* 18 d.) (In affirming the second, Plato is rejecting

the position of the atomists, just as in affirming the first, he is rejecting the position of Parmenides.) It must not be supposed that for Plato the two principles exclude each other. Being is through relatedness; but in order that there should be a relation, there must be terms. Thus, relatedness presupposes distinctness. Conversely, the terms, as distinct from each other, exist in virtue of this relation of diversity. Moreover, communion is a property which makes distinctness possible; the individuals in the state achieve, each one, his own individuality, through the fact that they function as members of one state. Thus, division and integration are through one another; they participate in each other. "When a person at first sees only the communion of the many, he must not give up until he sees all the differences in them, so far as they lie in classes; and conversely, when all sorts of dissimilarities are perceived in a large number of objects, he must find it impossible to be discouraged until he has gathered together into one circle of similarity all the things which are related to each other." (*Pol.* 285 b, Loeb tr.)

In order, however, to distinguish the two phases of any entity, we will coin an appropriate terminology. We will speak of the *intrinsic* nature of an object to designate its character as something for itself, and we will speak of the *relational* nature of an object to signify its character of a being through other things. For example, the transcendence of God is his intrinsic nature, his enjoyment of his own self-sufficiency; whereas the relational nature of God expresses his relatedness to the Good and to the forms, and his immanence in the concrete world.

The principle of the One and the Many is external or internal, horizontal or vertical. We will take up each aspect in turn, beginning with the conception of the Many.

To say that Being is divided *internally* is to maintain that the really real is internally articulated, differentiated. Parmenides is wrong in his doctrine of the bare One. The

Many is the principle of the atomization of being. Given anything, it splits up (*Phileb.* 23 a, 25 a, 25 d; *Sophist* 257 a); nothing is ultimately unanalyzable. The real is a unity of parts. Thus, the created world is an organism; and the metaphysical situation is complex. The real is not homogeneous, or reducible to a single category; to formulate the real we must appeal to a plurality of categories, such as motion, rest, same, other, etc.

In the first place, the One is divided in the sense that any given being is analyzable into this, that, and something else, each of these being other — *ἕτερον* — than the rest. Thus, the real consists of entities which are diverse *numerically*. The distinctness, in the second place, is *qualitative*. The given being is divided into parts which are dissimilar — *ἀνόμοια* (*Theaet.* 185 b; *Pol.* 310 a). There is the passage from the like to the unlike. The given breaks into a many, each of which has a specific nature. The individuals in the ideal state are differentiated according to function — one man, one function. Does Plato intend to distinguish between numerical and qualitative difference? In all probability, he was not aware of the problem in these terms; we simply note the fact that he uses these two terms, *ἕτερον* and *ἀνόμοιον*.

Thirdly, beyond diversity of nature, the principle of the Many imposes contrariety of nature, — contrast. Thus the good man is one who unites opposites — gentleness with courage (*Pol.* 310 ff.). The real includes rest and motion; the philosopher combines enthusiasm with calmness; knowledge is both inspiration or ecstasy, and discourse. The royal art consists in the harmonization of opposites; the king is a weaver who blends the colors of opposing characters into one web; indeed, the royal web of truth is woven into one fabric through the reconciliation of a clash of standpoints. In sum, the principle of the Many in the One means that the real is atomized into members which are *diverse, different, and opposed* — *ἐναντία*. The division of the One into a Many is a step in the achievement of determinateness. The

continuum of the line is differentiated into a multiplicity of segments each of which bounds the other. Such a differentiation makes use of the factors of Being and Non-Being or Same and Other ; that is to say, within the One there are entities each having its own self-hood, each identical with itself, and other than the others.

Similarly, the principle of the Many applies to the realm of forms. We do not have 'form' in general ; we have this and that form,—black, white, horse, ox. And it is this diversity among forms which renders possible qualitative difference among particulars. Thus, there is a sense in which we have sheer diversity among forms, such that each form functions as a particular, other than other forms, e.g. whiteness other than roundness, or man. In this sense the realm of forms bears the character of the world of particulars. Each form is an *ἕκαστον*.

But there is a further aspect in the differentiation of forms. The One is not divided into the Many *at once* (*Phileb.* 18 a, b). A general class is divided into a definite number of subclasses ; a sub-class is divided into subordinate groups, and so on, until the indivisible—*ἄμνητον*—is reached (*Phaedr.* 277 b). There is, thus, an intermediation between Form, in general, and the many forms ; and the intermediaries are what we would today call universes of discourse. White and black are divisions of the genus, color ; round and square, of the genus, shape. Hence, we do not compare white and round directly ; we do not designate the first as the *ἕτερον* of the second ; we compare them only in so far as the one is a color and the other a shape. In sum, the totality of forms is an organized group, consisting of closed systems within the general system.

In the *Philebus* (17 a) Plato states emphatically that it is not enough for science to know that such and such a field constitutes a unity ; we must know in what respects it constitutes a unity. Knowledge demands detail as much as it demands generality ; it is specific. We must have the mo-

dalities of the unity, the 'how' it breaks into a plurality. There is diversity and division in the very manner in which the real is divided; there are the intermediary groups, particular classes by means of which the One is distinguished into the Many. Otherwise we have vagueness.

Not only is it necessary to *specify* the One; we must *particularize*. The formulae must be related to individual cases. The division must be carried on until particulars are reached. The physician knows his art of medicine when, beyond knowing that such and such drugs have such and such effects in general, he knows at what time, in which particular case, and in what particular quantity, the drug should be administered (*Phaedr.* 268 b, c). A pupil has mastered his rhetoric, when having learned what sort of speech will influence what sort of man, he is able to interpret his knowledge into particular terms, that is, is able to recognize this man as an instance of the type, and to deduce what particular sort of persuasion he should bring to bear in the particular case which confronts him (*Phaedr.* 272 a). Thus, knowledge is not genuine, definite knowledge unless it is both *specified* and *particularized*.

Conversely, we proceed from the Many to the One; we have synthesis as well as analysis. The world is not a sheer plurality, any more than it is a sheer unity. The city is not a chance collection of private interests, each one with its own particular end, and conflicting with the rest; it is a unity in which the various interests serve a common good under the guidance of the philosopher-king. The citizens are knit together by friendship (*Rep.* 428 d). Friendship is the factor which expresses the coherence of individuals into a whole. The demagogue serves selfish interests; the philosopher serves the public good; selfishness is of all evils the greatest (*Laws* 731 e); it is the resistance to integration and leads to destruction.

According to Protagoras, man is the measure of all things. This would mean that there is no common ground among

minds; that each mind is a private perspective with its private world and its private truths. Such a doctrine leads to the disintegration of knowledge. According to Plato, on the other hand, the standpoint of reason is universal, and minds meet on the ground of common standards. The principle of friendship, of rational persuasion, of discourse and of intercourse—which is nothing other than the metaphysical factor of *κοινωνία* (participation)—makes it impossible that the metaphysical situation should be conceived as a plurality of windowless monads—windowless whether in respect to desire, or to knowledge, or to causal interaction. Nothing exists *simpliciter*, by itself; the real is never a mathematical point, so to speak; it is a collection, a systematic unity of parts. To be, is to be a whole of parts. (Here we have the whole-part relation.) The real is a relational complex, a ‘syllable,’ to use Plato’s favorite figure. The many are ‘bound’ together; separately, they are without the restraining influence which each exerts upon the rest; separateness means excess, indefiniteness. But together they limit one another; they are tamed, bounded, and so achieve definiteness. Plato speaks of the *bond*—*δεσμός*—which connects the parts in a whole. The bond captures the indefinite multiplicity into a unity. The king is the bond of the city; reason of the appetites; God is the golden cord that binds all things (*Laws* 716 c).

The unity into which the bond brings the parts is a definite unity. Here we encounter Plato’s conception of proportion and harmony. Harmony is a notion concerning the problem of the relation of members in a whole. Each part is proportionate to each. Thus, the conception of modality comes in once more; unity is not anyhow. A whole is a ratio. The mathematical formulae are precisely the rules according to which a whole is constituted from the parts. And the numerical proportion constitutes the whole into an esthetic whole, with contrast and balance. We have here the notion of fitness. Thus, a tragedy is an appropriate

combination of parts so that they harmonize with each other and with the whole (*Phaedr.* 268 d). Finally, the parts are integrated so as to form a whole, a living being in which each element is responsive to each. Thus, the idea of life — ζωή — arises out of the general conception of the One in the Many. The created world in the *Timaeus* is a living being — ζῶον —; so is the eternal pattern. The universe is such that the parts derive their sustenance from the whole and their end is the good of the whole (*Laws* 903 b, c). Knowledge, too, is an organic structure in which each consideration bears on all the rest, and on the whole (*Phileb.* 64 b; *Phaedr.* 265 e). And it is possible from the clear apprehension of one idea to infer the entire system of ideas (*Meno* 81 d).

In the above discussion, we have taken the idea of the One as signifying relatedness, or participation; and the idea of the Many as signifying otherness, or distinctness. The exigencies of exposition have made it appear that the individual elements are prior to the whole. Plato himself occasionally speaks in that vein. Thus, he states that God created the soul by dividing and then binding the parts (*Timaeus* 37 a); but this is only a way of speaking. Elsewhere, he speaks of the whole as prior to the parts (*Phileb.* 29 c, d); essentially, for Plato, analysis and synthesis are coordinate phases of being. It is not true that the letters come before the syllable. The syllable consists of the letters, and the letters are abstracted from the syllable. The parts determine the whole, and the whole determines the parts (*Phaedr.* 266 b, *Phileb.* 16 c).

We will now proceed to consider the One and the Many as principles of external ordering. Consider the Many.

We are concerned with the principle that being is bounded. Given something, there is always a more; nothing exhausts being; nothing is everything. Whatever we designate, whether God, or the Good, or the Mixed Class, or the Unlimited, we leave something out. There is no ultimate total-

ity. The 'other' pervades the whole range of being. It may be objected that the Good (*Rep.* Bk. VI) is a totality inclusive of the whole of being. Our answer at this point can be of a general nature,—that in so far as there is dialectic for Plato, nothing is complete; whatever is, is transcended by something else, into which it moves. There is always contrast. Thus, to posit anything is to posit a world, which constitutes its metaphysical environment. This is the sense in which being is through non-being. Thus the Good has its other: i.e. the Forms, into which it moves. In short, every entity is delimited, marked off from the rest of being. Given an entity, there are the ἄλλα (others).

Let us now return to the wholes which are obtained by the integration of the elements. The principle of external differentiation consists in the fact that internal integration is subject to limitation; in forming the wholes, so much is included and not more. There is what is present, and there is what is absent. A whole is not a collection of all the beings; it is a bounded whole. As Plato says (*Soph.* 252 e), there is no universal communion of classes; there is inclusion of some, along with exclusion of others (except for the highest kinds, i.e. the categories, which participate in everything; *Soph.* 253 b, c), and any concrete thing is a plurality of beings—that is, an integration of parts; with an infinity of non-beings—that is, with an infinity of parts which are excluded. Every being is a congregation of elements inside a circle which is drawn upon an infinite area (*Soph.* 251 b, 256 e).

The correlative to limitation is self-hood. In so far as an entity excludes other things, it is itself. The real is autonomous, self-sufficient, adequate — *ικανός* —; that is to say, it is an individual.

FINALLY, we have external integration. Every being is surrounded by an ocean of non-being. The problem is posed of the relation of a being to the region of other things. The

doctrine of communion is that a given thing stands in a definite and essential relationship with its universe. The individual is responsive to his environment, acts on it, is acted on by it. Once more, we find emphasized the life and motion in the world. There is the interplay of the organism with the area of the 'others'—*ἄλλα*. In fact, there are two movements: the movement within the organism, among its own parts, and the movement of the organism as a whole toward its world, its adjustment, its adaptation—*ἁρμονία*, its receptivity to the environment in which it finds itself. The individual cannot be torn apart from the world; there is an inner life, but it is not complete; for its realization, it requires the air and sunlight of the world. Therefore, education for Plato is not *merely* something that one does to himself; it is also guidance, as when the man released from his chains in the cave has to be dragged up to the Good by someone else (*Rep.* 515 e); and, the ascent to the Idea of Beauty (*Symp.* 210 a) is carried out with the aid of a friendly daemon. There is in Plato the paradoxical doctrine of the soul as by itself, in its own solitariness; and also of the soul as friendly or befriended, teaching or taught,—the soul as *with* other souls. The right Soul cannot grow except in the right city; in short, the dependence of the soul is not only upon other individuals taken singly, but upon groups, upon the going life of the community, as manifested in its institutions, its government, its works of art, more particularly its intangible life, its habits and ideals. The fifth book of the *Republic* is an account of the degradation of the philosopher by the mob; the ninth book, of the degrading influence of a poet without moral purpose, upon the community. (Thus there is the dependence of the individual upon the community; and there is the dependence of the latter upon the former).

In short, the organism—the coherence of contrasted qualities which makes up the individual thing—cannot function in a void. But there is a limitation to the participation of the individual in his environment. There might be, for in-

stance, a dependence on the community so complete, that the person loses his self-identity, and ceases to count as an individual. This is the situation when the person uncritically adopts the opinions and conventions of the public and of his day. He is then only a receptacle; he has then no more reality than the reflection of an object in water; he is a shadow. He can be real only in so far as he is reason, that is, only to the extent that he persuades himself of his views by rational argument. Then only is he self-determining, then only is he a soul. The function of the social environment and of the friend and of the teacher is to guide the individual to self-discovery, to self-knowledge.

Thus, we should not speak of limiting the magnitude of the individual's dependence upon his world. It is rather a question of the *sort* of dependence. Shall the individual be a mirror to his world, an imitator, an actor playing roles written by the playwright, or shall the role of instruction consist in stirring the soul out of its torpor, of leading the soul out of the darkness of the cave, that is, of provoking the soul to become conscious of its love for beauty and the good, latent in it? And this is how the paradox of the dual life is solved in Plato. There is the interplay of the individual and the social life. Man is indeed both a political animal and an individual by himself. The effect of the teacher upon the individual should be such as to drive him into himself, that he may discover his inner life; thus, we have the soul gathering itself into itself (*Phaedo* 65 c); and conversely, the philosopher who has achieved a knowledge of his real self, and of the Good, is driven by the very nature of the Good as self-transcending to return to the community and to share in its labors and honors (*Rep.* 519 d). Thus, the problem of education is dual: on the one hand, it means transforming the inner life of the individual; on the other, improving the institutional life of the community.

An allied point is the following. In so far as the individual participates in the world, he transcends his individual-

ity and becomes his own other. This is the principle of the self-transcendence of things, such that the universals are present — *παρουσία* — in the particulars, the soul in the forms and in the body. Beauty imparts itself to things, and the Good, like the sun, radiates its light upon the world. God is in the world and cares for it, as a shepherd tends his flock. This is the doctrine of being as power. This relatedness of things is (a) causal, in the sense that an entity exerts or receives activity; it is also (b) formal; thus, the forms are in communion with each other; in other words, there are necessary *relations among forms*. There are genuine, non-tautological implications; we have synthetic relations *a priori*; knowledge is saved from being a mere affirmation of identity, that A is A. Plato's logic is a logic of synthesis. Thus, we have dialectic, which is the passage of one form into another. Further, inasmuch as the particulars participate in the realm of forms, it is possible to infer one fact from another; to speak in modern terms, induction is valid. (We know that snow will extinguish fire because we know that the cold excludes the hot. *Phaedo* 103.) Thus, communion is motion, as distinctness is rest.

To sum up, by virtue of the category of non-being, being divides into the many. By virtue of the category of participation, the many are integrated into wholes. Non-being re-enters the scene, functioning by way of limiting the membership of the wholes. Thus, every whole is bounded. Participation appears again as the principle that each whole is related to its environment. It is the jerky movement of the dialectic, combining both analysis and synthesis, both rest and motion — like the movement of the train, which proceeds, stops at a station, starts and moves again. As Plato says in the *Sophist* (249 d) both movement and rest are indispensable for discourse — and, we might justifiably infer, for the *world* of discourse.

IN the preceding pages we have dealt with the internal and external principles of analysis, in isolation from each other. Actually, they are concurrent phases of the real, and we must consider them in their joint operation. On the one hand, an entity is internally differentiated into parts held together in a unity. An object achieves self-identity by virtue of its securing a unity over its many. On the other hand, an entity stands in an external relation toward the rest of being; it achieves a self-identity through its contrast with other things. Thus, an object faces two ways: toward its own internal structure, and toward its total metaphysical environment. To quote the familiar passage in the *Sophist* (256 e) "everything has a plurality of being and an infinity of non-being." In respect of the first, it is self-complete, and at rest; in respect of the second it is in-self-sufficient, restless, passing into other things. These two phases are of unequal importance. The self-maintenance of the entity is, so to speak, a momentary interruption in its dependency, which is its fundamental nature. An entity forever points beyond itself: the particulars suggest the universals, the forms intend the Good.

Thus, an entity subsumes a many; and, in turn, is subsumed along with other entities into a higher unity. An entity is the unity of a many, and also is itself one among a many. A human being is a cohesion of parts (the soul—with its own three parts—and the body); and is a member in the state. Moreover, the parts into which the entity is divided are themselves wholes—*ἅλον ὅλων*—(*Tim.* 33 a) divided into parts; and the unity under which the unified entity is subsumed is similarly gathered up into a higher unity (*Sophist* 253 d). Since the series is finite presumably, there is a term attained in the ascent which is the final term, not subsumed under a higher unity; and there is another term, or rather a group of terms attained in the descent, which does not subsume a many (it is *ἄτομον*—atom, *Sophist* 229 d). At the two ends we have last terms, the one of which has only inter-

nal differentiation, and the other only external relatedness.

Correlatively, knowledge has a dual aspect ; as the grasp of a self-complete object, it is static ; as the grasp of an object in its self-transcendence, it is motion ; in the latter sense, to know X would be to apprehend it in the totality of its relations. The argument in the *Theaetetus* is an illustration of this point. The first half of the dialogue (143-187) is a demonstration that knowledge is not merely motion, and the second that it is not merely rest. Thus, (a) knowledge is not of a changing thing, because in changing it eludes the mind. To be knowable, an object must *be* something, and so must be at rest. And similarly, knowledge is not a process, simply and solely, for then, it would have no fixed character, and would not even be knowledge. Thus knowledge is of rest and at rest (b). The second part of the dialogue demonstrates the reverse ; knowledge cannot be merely rest, nor of objects at rest. Suppose we define knowledge as an analysis of a complex into its simplest components. Then the components, being unanalyzable, would be unknowable ; so the complex, too, would be unknown. The implication of this argument seems to be this : that we regard knowledge as a mechanical reconstruction, as a synthesis of completed bits of apprehension. Yet knowledge is no more a series of rests, than motion is (*Cratylus* 439 e).

More specifically, Plato points out that if knowledge is an analysis of a complex into its elements, then the complex must be either the totality of its parts, or a whole with an indivisible unity. If the former, then, since the elements are unknown, so would the complex be unknown. If the latter, then the complex, which is indivisible, is simply another element, and unknowable like the primary constituents. The fallacy which Plato has in mind would seem to be that we are defining knowledge as a mechanical rather than an organic system. We are conceiving of the perception of the elements as already there, antecedently to their ingredience in the complex act of knowledge ; and so we construe the

complex act in terms of the simple acts. Yet the simple acts do not exist save in their relation to the synthesis. There is no static knowledge, even of elements; all knowledge is relational; at each step of knowledge we are on our way to another step; and the first step is not really knowledge before the next step has been reached. In the *Theaetetus* the argument is negative; no positive results are given. Plato is still groping. What he definitely implies is that knowledge is not merely flux, and is not merely rest. That it is both rest and motion follows from our analysis of being, given above; in the sense that knowledge is of the object both as self-complete and as self-transcending.

So far we have been dealing with the One and the Many as an horizontal principle of order. Yet the parts in a whole stand not only in a coordinate relation with each other, like the letters forming part of a syllable; they also stand in an asymmetrical relation as higher and lower, which we will call the vertical order. To be ordered, it is essential that entities should stand in this asymmetrical relation, where some are higher, clearer, better, purer, more self-sufficient, more self-determining, partake of a greater reality than others. There is subordination as well as coordination. For instance, a state is a community of individuals engaged in specific functions in a common purpose; but also, a state is integrated in the sense that it is ordered according to classes: the philosophers, the soldiers, the tradesmen. And Plato says it is *natural* that there should be those who rule and those who are ruled (*Rep.* 444 d). This is the source of the notion of hierarchy in Plato.

There are innumerable applications of this in his writings. For instance, we have the ladder of knowledge and being (the divided line); the ladder of the good (at the end of the *Philebus*); the ladder of the *Eros*, starting from the love of the body and leading to the love of abstract Beauty; the ladder of politics (tyranny, democracy, oligarchy, etc);

grades of souls, and so on. For example, starting from the (1) Limit at the top of the series, we find the following in a descending order: (2) Highest forms or categories; (3) Ordinary forms (within these, there is a subsidiary hierarchy of forms of virtues, forms of natural things, forms of things that are made by art); (4) Concrete objects, or mixtures; (5) the world of images and of reflections; (6) the Unlimited. With reference to time, we have the following hierarchy. There are immortal things, and mortal things. Immortality is graded, in its turn. There is absolute immortality, such as that of God; and there is derivative immortality; this is the enduringness of the Gods and of the human soul, which is dependent on God. Thus, the soul endures forever; but this enduringness is not intrinsic to its nature, but is derived from God. Finally, there is the immortality which lies between genuine immortality and mortality. This is immortality by substitution (*Symp.* 208 a, b) whereby a creature dies, yet survives in its child. It is immortality through time and passage; immortality by self-reproduction. Reason is immortal in that it remains identical with itself forever; whereas, in the empirical self, nothing remains; there is the flow of consciousness, of feelings, of states, of opinions. But there is continual replacement and preservation of pattern. Thus, eternity casts its shadow across time. Finally, there is pure flux, unlimited change, passage pure and simple. In sum, we have (a) Underived immortality; (b) Derived immortality; (c) Immortality through substitution; (d) Complete change, with no repetition.

Given any conception, one cannot say offhand whether for Plato what it represents is good or bad, real or unreal; it depends on the level at which one considers it. Thus, is motion unreal? That depends. Yes, if one is referring to the flux of sense; no, if one is referring to the circular movement of reason. There is the hierarchy of motion in the *Sophist* (248 e): κίνησις, life, soul, mind. Does Plato condemn poetry? Again, one cannot answer by a simple yes

or no; there is poetry which expresses the impression of the senses; and there is poetry which makes use of the images of sense to convey something nobler,—abstract grace and harmony; finally there is the ecstasy of the vision of Beauty which transcends any symbolism. The function of the conception of grades of being in Plato's mind is that he does not wholly condemn anything; appetites are bad, but there is the desire of the good, which is good. Everything can be saved by being transformed and raised to a higher level of being. There is an imaginary line drawn from the Unlimited to the Good; the rank of the mixture is determined by its place on the line. The criteria of whether an actual thing is higher than another are its relative purity, abstractness, and fixity; thus, spirit is higher than appetite, and reason higher than spirit. It is really a question of the extent to which the mixture partakes of the intelligible world; and as the latter is the really real, 'higher' also means 'more real.'

The principle of subordination has sweeping implications for Plato's thought; it applies primarily to the ultimate metaphysical factors. All these factors are relevant in the constitution of the real; nevertheless, they are not coordinate. The *ἄπειρον* is ingredient in the total situation, along with God; but there is an asymmetry; God is more real, more effective causally, than the *ἄπειρον*. The receptacle and the patterns are both relevant in the creation of the world; but the status of the two is not the same. The principle of the complexity in the metaphysical situation, which we noted at the start, must be now qualified with the principle of asymmetry. The complexity is not anyhow; it is not, so to speak, on a flat surface; there is the *overloadedness* of the intelligible factors, as compared with the mixed class and with the receptacle. This is the relevance of the principle of Value. The sense in which the Good is prior to all things is that it establishes this asymmetry in the nature of the real. The world is not merely rational; that is, the world is not understood merely in being *described* as consisting of this or

that; it must be *evaluated*. Thus the Good is the defining notion of vertical order. To proceed with more illustrations of the asymmetry at the root of things,—the world is both One and Many, but the One is more real than the Many; the real is both Being and Non-Being, but there is an overloading on the side of Being. Thus, there is no democracy in the metaphysical situation.

If we then arrange the world in this asymmetrical order, we will see the principle of the One and the Many operating again. Take the Many. Plato says that, in knowledge, we should not descend *at once*—*εὐθὺς*—from the One to the field of particulars (*Phileb.* 7 a ff.). One must go step by step. In short, there is multiplicity in the vertical order; there is a continuous series of steps between the intelligible world and the receptacle. We are confronted now with the central notion of the “between,”—*μεταξὺ*—the intermediary, the *δαίμων*, whereby the hierarchical set is established as being complex. Plato insists that knowledge is achieved when, instead of proceeding *immediately* from the Many to the One, we grasp the intermediary steps. Science is not constituted by the simple application of a general concept to an indeterminate plurality, but by a gradual descent through intermediate concepts and definitions. Conversely, the particulars are not knit into a unity, *simpliciter*, but via species, which, in their turn, are grouped under higher classes. The conception of asymmetry is thus qualified by the principle of *continuity*. Such continuity, however, is not one where between any two steps there is a third. In the latter case, the number of steps would be infinite, that is (from the standpoint of Plato) indefinite. The number of steps is finite; also, the interval between any two steps is definite. Thus, a graded being occupies a definite position in the scale.

The concept of continuity is the defining principle of science; for science is the study of genus and species.

The principle of continuity is a consequence of the principle of *plenitude*. The various intervals between the Limit

and the Unlimited must all be occupied, because to leave them unoccupied is to fail in respect of completeness—*τέλειον*. Thus, the existence of entities of all degrees of reality; the very existence of the Mixed Class, that is, of the concrete world; and the hierarchy of forms in their various degrees of specificity; finally, the possibility of rational discourse conceived as classification—all these depend on the principle of plenitude, which legislates that there should be not only the best but also all the lower degrees of value. In fact, the principle of plenitude is the foundation of the complexity of the Real, of being as many. In its turn, plenitude issues from the principle of *perfection*. A world which were not complete would not be perfect (*Tim.* 33 a, 39 e, 41 c). At this point, the factor of the Limit passes into the factor of the Good and becomes indistinguishable from it.

It will be noted that the various steps in a hierarchy participate in each other and so form a unity. (This is the principle of integration operating in the hierarchical order.) The intermediary is not only a principle of separation, but also of relatedness; it connects the two factors between which it lies. The between is a mediator; it is an activity of interpretation. Thus, the *Eros* conveys the prayers of men to the Gods, and interprets the desires of the Gods to men. The *Eros* moves in both directions, from the lower to the higher, and conversely. It is both ascent and descent. The philosopher proceeds from the Cave to the Sun of the Good, then he descends back to the cave, and grasps the shadows in their real nature as shadows, through the fact that he has seen the real objects outside. In knowledge, ascent is generalization, integration, abstraction; descent is application of the general to the particular, interpretation of the abstract pattern in terms of concrete instances. Thus, movement is from both ends. The creative activity of God is a movement from the eternal in the direction of the temporal. There is also the reverse movement; the mortal has an appetite for the immortal.

Plato's theory of the intermediary as the interpreter may be illustrated from his general conception of human nature. Man is a cohesion of parts of different rank. Reason stands highest in the scale ; it links man with the realm of the divine, because it is immortal. Next lower is the mortal soul, which lies between reason and the body, and transmits the commands of the first to the second. Having affinities with both, the mortal soul is drawn in contrary directions,—by its mortal nature, it is drawn to the body, but owing to its spiritual nature, to reason too. The mortal soul is subdivided into two parts: spirit and desire. Spirit—*θυμὸς*—is a mean between reason and desire ; it is that part of man which, though not rational, obeys reason, and carries the commands of reason to desire. Spirit is reason in its character as effective in the realm of mortal things. Desire is the bodily part of the soul ; it is the link between soul and body. Finally the body is the link between man and the world of circumstance ; the soul becomes effective upon physical nature through the intermediation of the body ; conversely, the surging irregular movement of the receptacle threatens to submerge the soul, by its influx into the body. Thus, the body is the ground on which the soul and the forces of circumstance engage in battle. At first, the irregular movement overwhelms the soul, confuses it, makes it forget its purpose. That is why the soul recoils from the world ; it is afraid. But with maturity, the soul grows stronger, recovers itself, recovers its control over the body, and is able to use the body as a vehicle through which the chaotic external forces are canalized and made subservient to the purposes of the soul (*Tim.* 43-44). To sum up, man is a hierarchy of beings, organized so as to serve as mediators for each other. At one end, beyond man, is God ; at the other, below him, is flux ; he is the link between the two, for he has a part that is divine, and a part that is mortal and indeterminate. He is drawn by two forces, vacillates between them, and his fate is uncertain. By producing works of beauty out of inanimate nature, he is fulfill-

ing his role as an interpreter of God to the created world.

Since the Gods, too, are souls, they may be added to the ladder. The hierarchy of souls, as thus supplemented, would be as follows: (a) *Theos*, or God as theoretical activity; (b) the Demiurge, or God in his practical activity; (c) the Gods, or created creative souls of the stars; (d) the world-soul; (e) human reason; (f) spirit; (g) desire; (h) the souls of animals; (i) the souls of plants (i.e. the soul as merely vital activity); (j) the purely inert, yet orderly, motion of inanimate bodies; (k) the chaotic movement in the receptacle.

The hierarchical order has a dual character; it is an ontological hierarchy and a causal hierarchy. We began with the consideration of the first, whereby the real is arranged in a graded series. But now the members of the series are seen to stand with reference to each other in the relations of activity and patience, as cause or effect. (This is the aspect of *δύναμις* once more.) The higher member in the series is a cause for the lower either as productive of it, or as controlling it. The ontological order comprises a series of creatures; the causal order, a series of *creators*. The cause produces a cause, and that another cause; for, since the effect is an image of the cause, it must itself be a cause, though of a more restricted effectiveness. Thus, we have a hierarchy in which there are degrees of creative power. Also, creation is not direct but mediated. God creates the Gods; the latter create the mortal soul; and the mortal soul regulates the body. It is probably this conception of the causal hierarchy which gave rise to the doctrine of emanation in Neo-Platonic philosophy.

The hierarchy comprehends three types of beings. At one end, we have creators who are not creatures: God, and the Limit. At the other end, we have creatures who are not creative. In between, we find beings which are both creatures and creators; and this is the world, intelligible and concrete. This world is the mean between the highest and lowest. One may hazard the theory that the world of dis-

course lies in the region between the two limits of the series — the upper limit being the Good, the lower being the indefinite multiplicity. We understand any members in the interval in that we classify them under a higher group and divide them into a lower group. But this is impossible for the two ends. We cannot reach the particulars by proceeding from the genus and increasingly qualifying it through further specification. The rabble of particulars is ultimately indeterminate, and, as Plato says in the *Philebus*, one must let go (16 e) and jump down, so to speak, into the chaos of particulars. Similarly, one may not reach Beauty by successive steps of generalization; Beauty is above words, and beyond concepts (*Symp.* 211 a); by rising steadily along the steps of the ladder, one reaches a single science of the utmost generality; this is as far as one can go by this method, and it is only the vestibule. Then, suddenly — ἐξαίφνης — (*Symp.* 210 e), one has the revelation of Beauty. And the Good (as discussed in the sixth book of the *Republic*) is beyond *ousia*; it is the source of light, the source of knowledge, and cannot be known — if at all — by the same categories by which it makes knowledge possible. Thus, Beauty and the Good are known by a form of ecstasy; similarly, the world of sheer particularity is known by a spurious ecstasy, as in a dream. Rational discourse would seem to lie in the region between the two extremes, and to concern itself with the world of the mixed.

We have said that the hierarchical order is both ontological and causal. Thirdly, it is valuational; it is an order of better and of worse. Here, the defining principle is the good; the rank of an element is established by the degree of its value. (See above, p. 157 ff.)

The togetherness of the higher with the lower gives rise to a more complex type of mixture. The higher inheres in the lower and controls it — διοικεῖ καὶ ἐνοικεῖ. The type of mixture that we had considered, so far, is constituted by the union of the Limit with the Unlimited. This union is brought about by the activity of the Cause. *Now, we have the fur-*

ther togetherness of the Cause of the mixed, with the mixed. Thus, the Soul (*Tim.* 35 a) consists of *ousia* which is a mixture of the same with the other ; this enters as a single element into a larger mixture, consisting of the same and the other and *ousia*. This higher type of mixture is the mixture of the Absolute with the derivative, of the Creative factors with the creature ; and expresses the immanence of the intelligible in the sensible order. We have here a mixture in the second degree : the mixture of the mixed with the cause of the mixed. Possibly it supplies the answer to the problem of what is really real, for Plato. If we contrast the Limit with the Mixed Class, the Forms with the concrete objects, God and the world, and ask what is more real, it is clear that the Limit, and the Forms, and God are more real than the sensible world. But this answer does not dispose of the problem. One might raise a different question : what is more real, the intelligible realm as such, or the intelligible along with the sensible ; God as such, or God with the world ; the best as such, or the best, with the lesser good ? Then the answer may well be that the latter members of each of these pairs are the more real. (Thus, the account of the Good at the conclusion of the *Philebus* is one in which the Good includes both Measure and all the lower forms of the Good. In short, one might consider the hypothesis that for Plato, the highest good and the most real is the mixture of the primordial factors with the mixture.)

By combining the two principles of horizontal and vertical order, we are enabled to depict order by the figure of a pyramid, along which one may travel either horizontally, or up and down. There is a complexity of levels of being ; and within any given level, there is a complexity of coordinate members of a group. Moreover, the hierarchical and horizontal orders participate in each other. The bond which joins the parts in a syllable is 'higher' than the parts ; correlatively, a hierarchy involves the notion of sheer complexity, manyness.

We may now sum up the preceding discussion. To be, is to be limited, to be ordered, to form a system. Thus, as Plato says in the *Timaeus* (30 c) the intelligible world is a living thing, an organized whole. (Analogous is the notion of Measure which Plato lists as first in the hierarchy of goods (*Phileb.* 66 a); measure, lacking which, no actual thing can endure (*Phileb.* 64 e). Measure is order as the formal requisite of being. And this is the sense in which being is number, namely, an orderly arrangement in which each element occupies a definite locus in the system; thus, an entity is rendered determinate by the coordinates of the system. Of every being we can say where it is in relation to the whole, what is the interval separating it from the other beings (*Phileb.* 17 d), and how real it is, that is, what level it occupies in the hierarchy.)

We have said that according to Plato, the real is both One and Many,—these two being conceived as principles of either external or internal ordering, and of either horizontal or vertical ordering. The conceptions which have emerged during the analysis of the Limit are, then, One and Many, Whole and Part, Essence and Instance, Genus and Species, Being and Non-Being, Same and Other, Like and Unlike, Motion and Rest, Diversity and Contrariety, Communion and Separation, Proportion and Harmony, Continuity and Plenitude, the Bond, the Intermediary, Hierarchy. (It is evident that in this list there is overlapping and omission.) These are the defining notions of the Limit. The question arises whether they are forms. They must not be construed as analogous to forms, because they are the principles of the being of the forms and of their ordering. Plato uses the phrase: greatest kinds; they are superforms, categories. They are the principles determining which forms partake in each other and which do not (*Soph.* 253 c, d). No form mingles with all the forms, but the categories do, because they are the principles of the ordering of the forms; thus, every form exhibits being and non-being, relatedness and distinctness. What is,

then, the rank of the categories? We venture to suggest that they are the intermediaries between the Limit and the Forms. The Forms are the 'many' into which the Limit breaks up; and the categories are the link between this One and the Many. They are the modalities of the Limit; the 'how' whereby it constitutes the forms. We thus have the Limit, the Categories, the Forms, arranged in a pyramidal order; they, too, taken together, exhibit the principle of the One and the Many.

Our definition of dialectic must be now amplified. Dialectic, on the one hand, is a knowledge of what forms mingle together and which do not; on the other, it is a knowledge of the categories which regulate the mingling, and a study of the ordering of the categories among themselves, both as distinct from, and as related to, each other. Thus, dialectic entails two aspects, the material and the formal. The material aspect is the science of the classification of the forms. The formal aspect is logic, namely, the recognition of the categories basic to the classification of the forms and to all thought. But these two are not separate phases. Dialectic is the union of the formal with the material; it is the study of the categories as the 'how' of the mutual relations among forms and among things; or again, it is the science of the interrelations of the forms, as illustrating an abstract, logical order. Likewise, a man who knows his language well, is not someone who knows the language, and also knows the grammar of the language. He is one who knows his language grammatically.

The Limit, impressing itself on the Unlimited, elicits the mixtures which are the actual entities. The merging of everything with everything in the Unlimited is converted, by the intervention of Non-Being, into a set of distinct objects, each having its own nature. The union of the intelligible with the sensible results in the creation of a world of orderly sequences. As the forms have communion with each other, so the concrete things, by virtue of their participation in the

forms, enter into determinate and constant conjunctions with each other, or as we should say, obey laws. (Thus snow extinguishes fire, because the essence, cold, excludes the essence, heat. *Phaedo* 103 d, e.) But the receptacle is refractory and is never wholly subjugated. God introduces order into chaos, *as far as possible*. Thus, there is a limitation to the intelligibility of the world; there remains an element of objective chance. Our laws can be no more than statistical.

THE One and the Many are principles of thought as well as of being. The *locus classicus* is *Philebus* 15 d. "We say that the one and the many are identified by reason, and always, both now and in the past, circulate everywhere, in every thought that is uttered. This is no new thing and will never cease; it is, in my opinion, a quality — *πάθος* — within us which will never die or grow old, and which belongs to reason itself, as such." (Loeb tr.) Thus, the One and the Many are innate affections or directions of thought; to think in these terms is a timeless quality of reason. Further, the One and the Many are notions which are present in every single act of the mind; to think at all, or to utter speech, is to refer to an object as a One and a Many. Both analysis and synthesis are directions of thought, and they operate together in any given act of knowledge. Thus, the One and the Many are laws of thought as well as of being. There is a correspondence between being and discourse. For Plato, to maintain that the One and the Many are categories of thought, is not to exclude them from being; the categories of thought exhibit the nature of being, because reason reflects the real. Discourse itself is a being, a reality (*Soph.* 260 a) and therefore exhibits all the characters of being. We assert in thought the One and the Many because they are there in the Real. In short, the One and the Many are both subjective and objective; and knowledge is possible because of the correspondence of mind with the world.

AN INTRODUCTION TO THE  
*DE MODIS SIGNIFICANDI*  
OF THOMAS OF ERFURT

BY SCOTT BUCHANAN

THE vulgar distrust of the human mind's capacities reaches its acme when as at present it is persistently claimed that the discussion of symbols is futile and perhaps dangerous. God knows that there is sufficient evidence of confusion and failure in the intellectual enterprise in all its historical ramifications to justify recurrent discouragement, but vital needs in the course of time turn thought from empirical failure to the search for causes. There are many such turns in contemporary thought, and many of them have been occasioned by inflations and collapses in symbols.

Money is perhaps the most imposing and consequential system of symbols that may be noted in this connection. In at least one country the rediscovery of the symbolic nature of money has led to a complete reconstruction of other traditionally basic symbols, and in all countries recent discussion of money has been in large part a discussion of symbols. It is not hard to see that many practical affairs depend upon symbols in the case of money, but it takes sophistication both in expressing and in understanding ideas to realize that just as the success or failure in the manipulation of monetary symbols has practical consequences, so the success or failure in the manipulation of religious, poetic, and scientific symbols determines issues on all other levels of thought and practice.

In fact it is time, if indeed it is not a little late, to revise the modern historical prejudice which blandly dismisses the discussion of signs and symbols as antiquarian in interest or as sentimental and superstitious in result. The modern

period has been peculiarly concerned with its symbols. Beginning with Leibnitz there has been an almost continuous line of great minds that have thought a universal language a basic need for the continuation of western civilization. This is a public way of saying that they have found the symbols of their own tradition inadequate for their private intellectual purposes. The quality of Leibnitz's own work, its rhetorical tendency and its incompleteness, exhibits the inadequacy which all competent honest minds continually recognize in themselves.

It is a commonplace of intellectual history that the ambitious and monumental work in mathematical logic has come from Leibnitz's original discovery. Esperanto, Anglik, Ido, and a dozen other constructed languages are, as it were, by-products. The original aim of the International Library of Philosophy, Psychology, and Scientific Method to rally and organize all attempts at symbolic clarification has led for the present into an inventory of conditions which might make a basic English language available as a universal language of translation. Poets and literary critics wish to reorganize the world in some revolutionary way in order to supply supports for their poesis; scientists and critics of science vie with their literary fellows in emphasizing the need for symbolic sophistication. One might suppose that all this has come, as we are told in popular histories of thought, from the scientific and industrial revolution; we have a new world that needs new symbols for its thought and action. But I believe this would be only the reiteration of a comforting opinion, and I am concerned at present with the somewhat pessimistically diverging opinion that we have lost a world because we have neglected the kind of symbolic sophistication that goes with our permanent tradition.

Whatever be our opinion about the possibilities of examining our own mental processes, there is the historical fact that for a long period of time, from Plato to Duns Scotus, intellectual discipline consisted in special training in the con-

tinuous radical examination and revision of human usages with symbols. I have for some time been concerned with the reconstruction of these disciplines, namely the liberal arts, and I should like to report findings which I hope may be deemed evidences of progress. I shall dispense with argumentative proofs and proceed with exposition that I hope will contribute to my own and the reader's sophistication.

It would be a great mistake to suppose that the liberal arts and the corresponding sciences at any time reached canonical formulation and that therefore all we have to do is to find a document or even a group of documents and appropriate its doctrine. This would be multiplying on a large scale the common error concerning Aristotelian logic in which the *Organon* or some later work of an Aristotelian is taken as revealed scripture, containing not only truths, but all truth. It is true that the division of the liberal arts into the traditional trivium, grammar, rhetoric, and logic, and the traditional quadrivium, arithmetic, geometry, music, and astronomy, is attributed to Martianus Capella, and that these divisions were developed by Donatus, Priscian, and Boethius so that masters of the liberal arts from then on constantly refer to their works, but this is only an example of that peculiar combination of piety and controversy that makes medieval culture appear to us all of one piece; we ape the style when we bless our own opinions by calling them Cartesian, Kantian, or Spinozist. There is an equal error, due in the main to the medieval historians, which claims that the liberal arts were as empty or vague in content as they are today when we speak of a college of liberal arts and sciences. The liberal arts functioned continuously and universally for at least a thousand years, and that means that, like most human inventions and traditions, they underwent radical and profound changes. In fact it is necessarily true of them that their natures and functions are changed every time a symbol is used, even when the use is repetitive. A plausible case can be made showing that they are at present unknown because they have themselves led their

devotees into materials and techniques which can operate freely only in the absence and ignorance of their medieval formulations. I think we must admit that the open denial and suppression of ancient truths has been the protection of a vital illusion, a matter of life and death, for modern science.

The immediately preceding paragraph is a warning to the reader who may wish to pass an adverse judgment on the liberal arts because of what I shall say of them in the sequel. I wish to make it clear that I am only dipping into a stream of thought, a stream that is truly Heraclitan in its flux and logos. Having given this warning, I shall use it as a licence to exploit a definite text which comes as near as any that I know to fulfilling the demand for a canonical statement of the liberal arts, namely, the *De Modis Significandi sive Grammatica Speculativa* of Thomas of Erfurt. This text has commonly been attributed to Duns Scotus, but is now generally ceded to Thomas of Erfurt, who is obviously a Scotist. Its footnotes in the edition of 1902 are mainly quotations from the writings of Duns Scotus. I shall not enter into the technical grammatical points of the exposition since they depend upon fine points in Scotist doctrine, but I want to use the general theme of the book in order to make observations on the ways of symbols, or the modes of signifying.

The book is late in the tradition of the liberal arts, and it therefore takes it for granted that there is an art of grammar which is the general art of using symbols. The modern should stop and think about this since offhand it appears to be either a bland generalization from linguistics without proper restrictions stated, or a commonplace about any human art as viewed by a somewhat whimsical poet. Actually the assumption is a very powerful generalization whose freshness for moderns should lead to the recovery of a radical insight. It points to a common dimension in all the human arts, calls it the symbolic dimension, and claims that there are comprehensive rules which apply with respect to this dimension in

all the arts. Viewed in another way it says that each human art, liberal, useful, or fine, has a grammar which must be known if the art is to be mastered.

It was the Greeks who discovered that knowing an art involves knowledge of radically different kinds, and it was a part of Socratic irony to suggest to the useful artist that the rules of his art depended on a science of which he was ignorant. But to turn a useful artist's attention to his proper science it is necessary to give him a liberal education, that is to show him the liberal arts and sciences upon which his useful art depends. So with grammar, which is both useful and liberal, knowledge even for practice must come from a grammatical science and this is the science of the symbols which the grammatical artist uses. It will be a general account, in terms of rational principles, of the common properties of symbols.

All this is assumed at the beginning of the *De Modis Significandi* and it is in this context that we should understand the first statements about signs as they are quoted from Duns Scotus. For instance he had said that "to signify is to represent something to the intellect," and in another place the Latin approaches our usage when we say "to signify is to give to understand." Then, becoming more technical and critical, Duns Scotus is quoted as saying that "signification is not the formal reason causing the concept in the hearer, but it is a certain preliminary disposition, on which by the collation of the intellect there follows the causation of the whole concept out of concepts caused by the parts." The chain of quotation finally reaches back through Duns Scotus to Augustine: "A sign is a particular thing which in addition to the species which it brings into the senses makes something else from itself come into cognition." These quotations set the problems for speculative grammar and start the process of disentangling the modes of signifying.

If we are to make proper use of them it is more important to recall what was thought and said before and at the same

time they were formulated than what has been thought and said since their time; their background is better than their foreground if we wish to escape the artlessness of modern epistemology. We must turn to the history of culture and the history of metaphysics.

The relevant theme in the history of culture is the use and criticism of the analogy as a form of thought and expression. The history is a long one and it can be divided into many plots which I shall not here try to take apart and put together again. It will be sufficient to point out three familiar fields in which analogies play important and explicitly recognized roles. One is poetry and literary criticism, another is mathematics and science, and the third is religious ritual.

Rhetoricians, both orators and critics, were very early aware of the patterns and effects of figures of speech. Beginning with the lowly simile and metaphor and rising through the moralism of fable and allegory, the orator's effectiveness in speech was measured by the height of his sublimity and the subtlety with which he achieved it. Out of such rhetoric in the Iliad and Odyssey there arose the great classic tragedies, whose elevations were equalled only by the comic depths of Aristophanes, his predecessors, and his successors. In fact among the Greeks the tragic and comic theatre with their co-operative figure-making mold and support the imaginations of all the members of society, both high and low, so that when a Greek thinks and talks for us it is as if we were listening to an extraordinarily brilliant pupil in a school of poetry. This is of course almost literally true of writers in certain periods of Greek life when the sophists had actually taught all the good writers to speak and even argue from analogy. The Greeks did not need the movies because their minds were both by nature and by nurture kinematic figures of speech.

But the dynamics of rhetoric needed and found harbors and anchorages for their symbols, namely, the Orphic and Eleu-

sinian mysteries. The story of the tragedy and comedy is often told as if the Greeks once had religious cults which in the course of time gave place to their own dramatic shadows, but it would be far better to run the drama and the mysteries in parallel, as a kind of living analogy between complementary arts. The mysteries were the ever repeated vital figures of life back upon which the plays of the popular theatre were ever throwing new lights. The plays were to the mysteries as the sacramentals are to the sacraments of the Church, and if one can believe the accounts we have, the mysteries outdid the tragedies in sublimity. In fact their aim as mysteries was to keep the devotee persuaded not only of the plausibility of the analogy they insisted on, but of the fundamental reality of the sublimest side of this analogy, real life taking second place as shadow unclear and incomplete. The drama gave the relations between the parts of human life; the mysteries gave the fundamentum.

So it always has been with rituals and literature even when one seems at odds with the other. In the modern age of freedom of thought and expression the figures between religion and imagination are as ironic as they were in fifth century Athens. Gods die, are torn limb from limb, are eaten, and rise again as the seasons roll around; or as Aristotle has it, things have beginnings, middles, and ends . . . no matter how roughly independent human beings hew them to fit their little dramatic understandings.

In between the tragic and comic analogies and their prototypes in religious ritual there always come the analogies of scientific understanding. Analogies may be used for elevation and edification, but they are also used for measurement, one thing by another, things by ideas. The essence of science, be it young just from the matrix of popular myths, or adolescent in those nurseries of nature which we call laboratories, or old and contemplative in the armchair, is comparison. The scientist places one thing beside another, one mark beside another, one idea beside another, and reads off the

formula. Formulae then become the interstitial tissue of thought; systems take form and grow into cosmologies. Almost as in the case of religion the figure of scientific speech finds itself the fundamentum and has whether it will or not to dictate the relations that shall be seen and understood.

The elements of these systems of things that invade the senses and move the intellect are simple enough, they are ratios, and even to the child's mind they cannot help but be seen as like each other. We see, and before we can think, we analogize. In fact it takes a great deal of thought to catch up with our sight and recognize that what we have seen is that A is to B as C is to D, and as soon as we have done that we doubt if it is true.

Here in a nutshell is the history of analogical thinking, and the ever present problematic situation that makes symbolic thinking and epistemology unsolved problems. It merits a little closer attention.

A is to B as C is to D is of course a very elementary analogy. It has two analogues of two terms each. More elaborate analogies may have many analogues or ratios and many terms in each analogue or ratio. In fact there is no limit to the size or discreteness of analogies. No one will therefore take it as over-simplifying the problem if I say that the signification of a sign is effective by virtue of the analogical relations it holds in such a pattern. This is in fact a very vague way of referring to a very complex situation, and it would be better to return to the simple case and be explicit. A is sign of C because A and C hold corresponding positions in the typical analogy above. Similarly B is sign of D. Similarly again, but also by inversion, C is sign of A and D is sign of B. But good analogies can be taken by alternation too, so that the one above becomes A is to C as B is to D, and then A is sign of B and C is sign of D, and vice versa. In fact it looks as though A could be sign of any other term in the analogy except D, and it may be illuminating to note that any term in the analogy may be sign of any other terms ex-

cept one, namely the diagonal term as one can see in the rectangular arrangement :

A	B
C	D

Of course there can be happy and unhappy analogies as the mathematician would say there are interesting and uninteresting theorems in any geometry, but all of them are analogies and help to make signs. We might also say that analogies help to make figures of speech, and we could show how the rhetorician and the poet get elevation and sublimity by skipping along the horizontal and vertical lines of signification in the analogy, but it is more important here to follow the sign-making theme, to the end of seeing how things come to move the intellect.

To Thomas of Erfurt, who is directly concerned with the patterns and significations of Latin grammar, a sign is a *vox*, a *flatus vocis* if you like, and he recognizes the analogical context by pointing out that there are active and passive modes of signifying. When A is the sign of C, A is an active mode of signifying and C is a passive mode of signifying corresponding to the activity of A. In other words, in traditional words, A has an imposition, it is imposed or imposes itself on C; that is its first imposition. If C becomes the sign for A, then C is active to A's passivity and C has a first imposition.

But as soon as this is recognized and formulated, it entails the consequence that A as active mode of signifying has a second imposition, namely, its consignification with B. A is an effective sign only if it acts in a context with B, which also is a sign, in this case, of D. A's second imposition therefore involves B and its impositions which are similar to A's, and with respect to second imposition mutual or cognate with A's. Caught in this context A becomes in the language of Thomas a *dictio*, a manner of speaking, and with special reference to its organic context B, A is a part of speech, *pars*

orationis. All this is a matter for grammar to investigate, but there remains one further step in which A is enabled to become a term (*terminus*). In order to become a term it must move the intellect.

To recapitulate in slightly different terms: A *vox* becomes a sign as soon as it has an imposed context in an analogy. The analogy gives it two impositions, one as a name or sign of a thing, a first imposition; and another as it consignifies together with another sign in the same analogy, as a part of speech. Grammar as a science is concerned with exploring and formulating the analogical modes of these two impositions.

One of the first things that appears, and that cannot be denied for long, is that a sign functioning in an analogy has a kind of generality. It may be the name of one thing, but it is also the common name of several things, and as investigation proceeds, it becomes very difficult to find a sign which can be called a proper name. There are several ways in which this fact appears and several ways in which it can be handled. It may appear when the thing to which the name applies changes and we still wish to call it by the same name; or it may appear when we have named one thing and another thing turns up which is so like the original that it appears simply arbitrary to withhold the name from it. One way of handling this fact is to recognize that the name is not the sign of the thing itself but of one of its properties which may be common to many things; another way is to insist on the validity of the proper name and make up other proper names as occasion demands. These two ways obviously lead respectively to the realistic and nominalistic extremes in the liberal arts, and, we may add, to their traditional absurdities. The traditional way of following the argument without fear of absurdity is to distinguish between the intentions of terms, but before we do that it may be well to return to the analogy and see how the same problem arises there in a subtler form.

Signs invade the senses in analogical patterns; they march

in rank and file. Although it incurs the risks of over-refinement it seems wise to make one of those distinctions which has brought obfuscation upon the middle ages. Signs marching in rank and file make analogies; analogies can then be separated into signs on the one hand and ranks and files on the other. The analogy has form and matter, proportional form and signate matter. As the signate matter invades the senses the proportional form moves the intellect, and it is the composite of form and matter that makes a full blown symbol. This is all very neat and very tempting as a solution of the symbolic or epistemological problem, but it calls for a little careful scrutiny, and it may take us a long way. In fact it takes us to metaphysics.

One sort of scrutiny would lead through the psychological labyrinth to see how it is that the senses and the intellect conform so nicely to the analogical combination of form and matter. This is actually not so difficult if we accept the historical doctrines in rational psychology. There are the special senses that receive sensible species; there is the common sense that combines sense departments to give us round colored fragrant rough objects. There are memory and imagination that take these objects to pieces and put them together again according to their possible combinations, and finally there is the intellect that extracts the intelligible species. But all this becomes merely an account of solipsistic reverie if we raise the question of truth and error, or even of mental health and disease. The psychological solution is question-begging, as we have many occasions for knowing, and it is a very unusual modern epistemology that does not consist wholly of a merely psychological restatement of the problem.

In view of this I have taken recourse to the relevant historical metaphysics which seems to throw a tenuous but vivifying light on both symbols and psychology. This is not the metaphysics that goes pontifical as soon as it puts two sciences together and begins to ascend balloon-fashion to cosmological heights. It is the metaphysics that remains after the balloon

has collapsed. Beginning with Plato it has been recognized that there are peculiar terms that are used in all subject-matters both in construction and criticism, but that they themselves never get proper criticism and are left over, often in paradoxes and surds, at any rate as fashionably undefined. Plato collected those that he found in the *Sophist*, and they became the heritage of the metaphysical tradition which has been added to and revised ever since. Toward the end of the middle ages the list had settled down to six transcendental predicates, as they are called. Plato had had four and their opposites, Being and Non-being, One and Many, Same and Different, and Rest and Motion. The medieval six are Being and Non-being, One and Many, Good and Evil, True and False, Universal and Particular, the last two serving as reciprocal opposites. They have peculiar properties which exclude them from the company of well-behaving scientific terms. First, they are predicable of anything and everything. Second, they are predicable of each other reciprocally. Third, they are predicable of themselves. Finally, they are predicable of their opposites. These properties, if recognized, break all the rules of modern logic and most of the conventional rules of ancient logic. Just as the algebraic transcendentals do not make good roots of equations, so these transcendentals do not make good terms in rational propositions. Nevertheless, they are unavoidable even in the best discourse and they carry almost too much significance.

Metaphysics consists in all the possible propositions that result from bringing the properties of these transcendentals into play in serial and reflexive predication. For instance the metaphysics of symbols should consist in the successive predication of these terms to whatever terms turn up in the discussion of symbols, and the exhaustive dialectical investigation of any transcendental propositions, that is propositions whose only terms are transcendentals, that occur in the discussion. This is not the place to embark on any such metaphysical venture, but we have already turned up Universal

and Particular in a fairly important connection, and it may be illuminating to explore the transcendental verse which is built up of propositions that result from predicating these transcendentals of all others and in turn predicating other transcendentals of these. Thus, using the Latin words :

Ens est res  
Unum est res  
Bonum est res  
Verum est res  
Res est res  
Aliquid est res

Non-ens est res  
Multa est res  
Malum est res  
Falsum est res  
Aliquid est res  
Res est res

Res est ens  
*etc.*

Ens est aliquid  
Unum est aliquid  
Bonum est aliquid  
Verum est aliquid  
Res est aliquid  
Aliquid est aliquid

Non-ens est aliquid  
Multa est aliquid  
Malum est aliquid  
Falsum est aliquid  
Aliquid est aliquid  
Res est aliquid

Aliquid est ens  
*etc.*

I have put down so many of these transcendental propositions in order that the reader may, by recognizing in them certain metaphysical problems already familiar to him, get the feel of this rather unusual philosophical style. Incidentally the translation will remind him of his own experiences with Latin grammar and suggest the value of the grammatical approach to philosophy. I have not put down all the propositions that belong to this verse because, as I said before, this is not the place for an exhaustive exploration. In fact I shall consider only the last two propositions in each group above; it will be noted that in accord with their opposition they are repeated.

The nearest approach in modern times to an analysis of these propositions is of course to be found in Hegel and the neo-Hegelians, particularly Bradley and Bosanquet, but this is not surprising since these propositions are primarily dialectical, and also since the chief contribution of idealism to modern

thought has been the indirect light it has thrown on symbolic problems. But there are certain advantages to be gained by turning from these modern schools and becoming scholastic, in one of the many radical senses of the word. For instance there are advantages in using Latin as the basic philosophical language, some of these advantages being similar to the gain in precision that we get from mathematical notations, and some of them being similar to the power that poets get from wide ambiguity in language. It will be noted that the terms in these transcendental propositions are the terms that we most frequently meet in reading medieval Latin philosophical texts, and that they are on the whole the hardest to translate. As a matter of fact they are the most important terms in the basic metaphysical discourse of the middle ages, and it may be that their untranslatability accounts for the apparent failure of nerve in modern metaphysical thought, which really amounts to a break in the tradition of speculative thought.

Whatever be the truth in such historical remarks, let the foregoing be a warning and an apology for the following expansion and interpretation of the part of the Re-ali-verse most relevant to symbols.

There are four transcendental propositions that I wish to consider :

Res est res  
 Aliquid est aliquid  
 Res est aliquid  
 Aliquid est res.

Experience and practice in interpretation show that the first two propositions have the force of existence postulates in mathematics. They assert that (1) universals exist and (2) that particulars or individuals exist. Together they balance the apparently divergent claims of the realist and the nominalist and rule out any metaphysical solution which is not sufficiently dialectical to admit both. This has a direct bearing on the distinction of form and matter in signs or symbols and on the distinction of intentions that follows from it.

In the analogy we noted that there is proportional form and signate matter, and these are to be identified respectively with universals and particulars. The doctrine is formulated in the liberal arts as the distinction between the intentions of terms. Any sign that becomes a signifying symbol has first and second intention, a complex reference to many possible particulars and a simple reference to one universal. In other words both *res* and *aliquid* are predicable of anything signified. In fact it is by virtue of the truth of these predications that signs become terms in the full sense, that is terms with two impositions and two intentions.

The other two transcendental propositions that I wish to consider are the most dialectical propositions in this verse, since they are reciprocal predications of opposites. I shall not enter into the sound and fury that arise from the simple direct interpretation which identifies universal and particular except to remark that a term as defined above, is a very tempting example of what Hegel meant by the concrete universal. I shall rather retreat to the more complicated interpretation which bears on the form of the analogy.

If we take these propositions separately in their first and second intentions, they will say some very pretty things about analogies. In place of universal and particular I shall use simpler English words more relevant to analogical structure, to wit, same and other. *Res est aliquid*, taken in first intention, says that things that are the same are other; *aliquid est res*, taken in first intention, says that things that are other (different) are the same. These two propositions together state in the basic metaphysical language that part of the theory of the analogy which says that there is at least one genus to which the terms of a ratio belong in common, or that the terms of a ratio must be homogeneous. The converse of this says that in any genus there will be at least two distinguishable species with some degree of relatedness or relevance to each other. These propositions then give the metaphysical basis for the hierarchies of genus and species

that we find in the medieval logics ; I believe they have the same bearing on the calculuses of classes and relations in modern logics.

With an assumption that I shall not attempt to justify here we can pass on to the second intentions of these propositions. I wish to assume that determinate otherness within the genus-species hierarchy determines relations and degrees of relevance. This appears to be a necessary assumption for the theory of analogy, and could be derived from other transcendental verses, but for obvious reasons I cannot argue the point here. *Aliquid est res*, taken in second intention, says that otherness or othernesses are the same ; *res est aliquid*, in second intention, says that samenesses are other. With the assumed correlation of otherness and relatedness, these two propositions state the metaphysical basis for the validity of the analogical form. The first says that ratios, which state relations between others, may be equal or similar. The second says that there are distinct degrees of equality or similarity between various ratios.

I said earlier that I thought this metaphysics was a better basis for the validity of analogies than psychology and epistemology, that in fact this metaphysics bases not only analogy but also the psychology and epistemology of analogies on a firm foundation. Perhaps agreement on this point is too much to ask of a modern audience. Sympathetic insight into transcendental propositions requires a retreat and detachment from positions that are vital for a modern to hold, and these propositions seem like childish verbalism or over-tolerant mysticism to the modern critical mind ; I know this from my own experience. Let me ask something less : I have found analogies too verbal and too mystical for clear and happy analytical treatment, and I have found with pleasure that these transcendental propositions uncover the crucial points and make them intelligible ; I invite attention to them for that minimum purpose. Self-evidence has been claimed

for this kind of metaphysics; understanding is an important step in the direction of appreciating self-evidence.

Analogies are often avoided on the ground that they are too easy to find and too difficult to control. The preceding discussion gives the reason why this is so. Every entity has some degree of sameness and difference with every other entity; the analogical form will therefore be universally tolerant, and since analogies are not always, in fact seldom are, explicit there will be no immediate possibility of checking them for truth or falsity as there often seems to be in the case of propositions. Therefore analogies can be found anywhere and apparently can be irresponsibly expanded in any direction.

On the other hand there is a felt distinction at least between thin and thick analogies which we exploit when we attack other people's analogical reasoning. The basis for this appears in the notion of kinds of otherness and sameness, or degrees of relevance. There are two hierarchies closely related to each other within which relevance can be measured, the genus-species hierarchy and the hierarchy of relations. The genus with respect to which the two terms of a ratio are said to be homogeneous has some determinate logical distance from the special terms. This distance may not be known explicitly, but it is rationally determinable by filling in the relevant intermediary terms. Similarly the relational context for a given analogy has a determinate structure although the mere analogical statement usually suppresses its explicit formulation. This is the basis for the vagueness usually attributed to analogies, but it is cured again by filling in the terms and tracing the multiple relations which are rationally there to be found.

This notion of distance within these hierarchies has a bearing on the fitness and clarity of symbols. If, as I have said, one member of an analogy signifies another along lines of signification between corresponding members of the constituent ratios, and the correspondence is determined by the

similarity of relations in these ratios, there will be a symbolic distance determined by the relational patterns in the ratios. Distance will be measured by the number of intermediate terms that have to be supplied to make the relational pattern explicit, and clarity and precision of reference will be a function of the measurability of this distance.

This character of distance within analogies can be viewed from another angle. With the intermediary terms not yet determined and therefore the distance and the determinate relations unknown, analogies appear to be viciously ambiguous. From an extreme rationalistic position this is pounced upon as the radical imperfection of all human knowledge, and we can only hope to clarify isolated small regions of it one at a time. From the metaphysical point of view, there is a somewhat different moral to be derived; from that point of view an analogy and therefore all symbols are systematically ambiguous in the mathematical sense of the phrase. They are multiple relational patterns containing the possibilities of progressive clarification and increasing determinateness for human knowledge. The rationalist who throws away an analogy because it has led him astray has thrown away the only marks by which he can find his way. It is a peculiar characteristic of the modern scientific temper that it moves forward by continually claiming to lose its way in this sense of avoiding its leading analogies.

Perhaps this is the place to enter a word of defence and warning. As far as I can see there is no reason to be drawn from what I have said for advocating at present or at any time a more extensive use of analogy. I have been describing what takes place of itself in human thinking without our knowing or controlling it. Savages and children think analogically, but that is because they have no better methods; a great deal of our own thinking is analogical for the same reason. It is a universal primitive unavoidable way of thinking because it is the very texture of our symbols; in that sense it is fundamental. Whenever we think about our sym-

bols we should take analogies into account. On the other hand analogy is not a substitute for argument, demonstration, or speculation, whether these be deductive or inductive in intent. Paradoxically enough, analogies are more fundamental than that; they are the ways we see both perceptually and intellectually. Like children they should be seen or seen through and not heard either in dispute or exposition. A great deal is to be gained, as in modern science, if the explicit use of analogy is reduced to a minimum, avoided, or ignored. As Aristotle says, the mastery of metaphor is a sign of genius, but he should have added that mastery by metaphor or analogy is a sign of immaturity. It is only in this respect that our fear of medievalism is wisely founded. In fact our own period may go down in history as the epoch of wild analogical thinking.

The cure for wild analogy is the critical analysis of analogy and the continual watchful scrutiny of the modes of signifying which are the subject matter of speculative grammar. The work of Thomas of Erfurt to which this essay may be considered a partial introduction for moderns presupposes the kind of familiarity and sophistication about analogies that I have been laboriously expounding, and the main burden of his exposition is to show the routes of signification through the terms of Latin grammar. The basic terms of his exposition are, on the strictly grammatical side, nouns and verbs, and on the logical side the transcendentals and the Aristotelian predicables, essence, genus, species, property, and accident. Nouns are divided into two kinds, substantive and adjective; the twenty-four kinds of the latter are impressive and exhaustive, showing the many kinds of dependent existences that are reflected in the Scotist use of Latin. The chief distinctions here are based on the serial nature of the modes of dependence on the substantive. The analysis of verbs follows a parallel pattern since the chief grammatical function of the verb is to compound the forms and matters which the orders of nouns have distinguished. Pronouns, adverbs, and

prepositions take their interstitial places. Finally there is a discussion of grammatical construction which approaches a kind of rhetorical analysis of the larger grammatical units.

I shall not go farther into these details, important as they may be, but I shall only point out one or two general themes that run through the exposition and use them to comment farther on the nature of symbols.

In the first place it seems to me that it is clear throughout that the subject-matrix about which the various propositions in the science of grammar are made is the analogical pattern that I have been describing. As I have suggested, the modes of signifying follow lines of signification that run as if horizontally or vertically between rows and columns of rectangular arrays of terms. It is through this analogical form as through a medium, that signs move the intellect. But it is insisted throughout that these forms of signification have their bases in the properties of things; active modes of signifying are directed to passive modes of signifying, and these are properties of things. The Latin is interesting, *proprietas rerum*, if we recall that *res* is in other contexts a transcendental, meaning universal. Both property and thing are to be taken in their broadest meaning: there are essential properties, proper properties, and accidental properties; things may be composite substances, formal substances, or merely universals. In the Scotist system these make a fairly complete list of the kinds of being, and the task of the grammarian is to show how the modes of signifying not only reflect these distinctions but also connect any particular mode in however complicated a grammatical manner with the appropriate basic property of a thing. Some things and their properties may be beyond the powers of human reason, and to that extent any system of symbols must come short of adequacy, but it is in the nature of things and in the nature of human knowledge that the system of signs shall be founded in the system of the things of reason (*res*).

But this claim raises doubts that have embarrassed modern

thought almost continuously since the time of Duns Scotus and Thomas of Erfurt. The origin and career of the Latin language, limited as it is both in space and in time, cannot be adequate to the scientific and philosophic themes of all time and eternity. In the case of any special set of symbols it seems that there must be not only a radical incommensurability of sign and thing signified, but also a component at least of vicious arbitrariness of construction which will fool the wisest of men if they take the burden of discursive thought seriously. Descartes and Leibnitz were seriously concerned with this difficulty and set the fashion of non-conformity and doubt for the modern period. Aristotelian logic came under the shadow of this doubt as soon as it was suspected that Aristotle connived to make his categories fit the grammar of the Greek language. At the same time there were attempts to save both Latin and mathematical notations by transposing them from the human tongue and the pen to more substantial mediums such as the lever, the balance, the astrolabe, and the wheel, where at least natural things could more obviously defend the symbols against merely human doubts. Also at the same time local European vernaculars took on the responsibilities of technical discourse, and of course a similar thing happened in literature, and religion. It is interesting to recall that Roger Bacon foresaw this welter of translation and vehemently pled for the study of grammar as the only stable basis for empirical science; his plea was in effect a plea for a new study, the grammar of nature. It is interesting also to note that the only breaks in the continuous exposition of the *De Modis Significandi* are made to answer objections which claim that various modes are fictional, a theme that runs regularly through modern epistemology. There is always the fear that symbols not only have too much distance from their bases in things but that they have lost their connections altogether.

Recently the rise of pragmatism and the fundamental critical work on the foundations of mathematics have almost

completed the circle of doubt and non-conformity to the tradition. The operationalists have returned unawares to a medieval doctrine that has been so long neglected and might have quieted doubts long ago if it had been recovered. This doctrine points out the fact that all things have operations as well as properties, in fact having properties entails having operations. This applies to nature and human nature, and the attempt to deal with anything without noting its operations is bound to be vicious. As far as human knowledge is concerned with empirical subject-matters, that is universally, knowledge of properties comes to beings below angels only with attention to operations. It is not necessary to go all the way with the modern operationalists and say that a thing is its operations to see the truth of the medieval doctrine. In fact the modern statement is a typical metaphorical slip.

Actually the operationalists are teaching us a lesson in speculative grammar; some of them even call it that. Nature is in operation and loses or distorts its significance for him who forgets or chooses to ignore its ways of coming and going. Man including his rational faculties is a part of nature and his signs and symbols operate and are operated upon. Operation is nature's and man's only way of keeping their temporal and spatial existences faithful to the eternal ideas from which they get their existence and their meaning. Human knowledge suffers when it neglects temporal things and their temporal properties simply because they are only shadows of eternity. Man himself is in large part only such a shadow and it is well for him to remember Plato's plea for a return to the cave, and the things to which he is analogically equal.

The place of operations in grammar are recognized in the terms imposition and intention. Things that are potentially symbols, and that includes for moderns many things besides voces, are pressed into service and have symbolic functions imposed on them, and they thus receive their intellectual intentions. As far as we know these are human impositions and intentions, the gifts of human operations, and man is act-

ing according to his nature when he performs them. Further there is no good reason to suppose that human nature is acting viciously or futilely when it acts symbolically. In fact there are many good reasons to suppose that man is acting in accordance with nature in a very profound sense when he thinks in his rational animal way. I have tried to give the best reasons that I know in the metaphysical discursus of this essay. Analogies and symbols are directed to the most fundamental and most general properties of things, their samenesses and their differences. The operations of their construction correspond not merely to isolated human nature but to natural things and their properties as they also operate.

It seems that we have lost our faith in our symbols largely because we have allowed them to go grammatically uncriticized so long. Grammar is a highly technical and specialized art. Furthermore it is a peculiarly complicated art because it requires us to use our symbols in criticizing them, and our use of them can be no better for this purpose than it is in their ordinary uses. On this account improvement must be gradual and probably slow. We are further hampered by the decay of the traditional grammatical disciplines in our educational systems. It is not always realized that the decay of our so-called classical studies entails an equal decay in our liberal arts, and that we are sending our students into the great complexities of natural grammar, in the laboratory, without much formal training in the more limited and formalized grammar of our own language, not to mention the so-called dead languages. In education it must be remembered that formal convention is the mother of nature, and that natural knowledge can be no better than our mother tongues make it. It is profoundly significant that our knowledge can be no better than our modes of signifying. We have a more extensive grammar than the medievals had to study, but we also have a more elaborate equipment to do it with, if we wish to use it.

# TRUTH BY CONVENTION

BY WILLARD V. QUINE

THE less a science has advanced, the more its terminology tends to rest on an uncritical assumption of mutual understanding. With increase of rigor this basis is replaced piecemeal by the introduction of definitions. The interrelationships recruited for these definitions gain the status of analytic principles; what was once regarded as a theory about the world becomes reconstrued as a convention of language. Thus it is that some flow from the theoretical to the conventional is an adjunct of progress in the logical foundations of any science. The concept of simultaneity at a distance affords a stock example of such development: in supplanting the uncritical use of this phrase by a definition, Einstein so chose the definitive relationship as to verify conventionally the previously paradoxical principle of the absoluteness of the speed of light. But whereas the physical sciences are generally recognized as capable only of incomplete evolution in this direction, and as destined to retain always a non-conventional kernel of doctrine, developments of the past few decades have led to a widespread conviction that logic and mathematics are purely analytic or conventional. It is less the purpose of the present inquiry to question the validity of this contrast than to question its sense.

## I

A DEFINITION, strictly, is a convention of notational abbreviation.<sup>1</sup> A *simple* definition introduces some specific expression, e.g. 'kilometer', or 'e', called the *definiendum*, as arbitrary shorthand for some complex expression, e.g. 'a thousand

<sup>1</sup> Cf. Russell, *Principles of Mathematics* (Cambridge, 1903), p. 429.

meters' or ' $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$ ', called the *definiens*. A *contextual*

definition sets up indefinitely many mutually analogous pairs of definienda and definientia according to some general scheme; an example is the definition whereby expressions of the form ' $\frac{\sin \text{---}}{\cos \text{---}}$ ' are abbreviated as 'tan---'. From a formal standpoint the signs thus introduced are wholly arbitrary; all that is required of a definition is that it be theoretically immaterial, *i.e.* that the shorthand which it introduces admit in every case of unambiguous elimination in favor of the antecedent longhand.<sup>2</sup>

Functionally a definition is not a premiss to theory, but a license for rewriting theory by putting definiens for definendum or *vice versa*. By allowing such replacements a definition transmits truth: it allows true statements to be translated into new statements which are true by the same token. Given the truth of the statement 'The altitude of Kibo exceeds six thousand meters', the definition of 'kilometer' makes for the truth of the statement 'The altitude of Kibo exceeds six kilometers'; given the truth of the statement ' $\frac{\sin \pi}{\cos \pi} = \frac{\sin \pi}{\cos \pi}$ ', of which logic assures us in its earliest pages, the contextual definition cited above makes for the truth of the statement ' $\tan \pi = \frac{\sin \pi}{\cos \pi}$ .' In each case the statement inferred through the definition is true only because it is shorthand for another statement which was true independently of the definition. Considered in isolation from all doctrine, including logic, a definition is incapable of grounding the most trivial state-

<sup>2</sup>From the present point of view a contextual definition may be recursive, but can then count among its definienda only those expressions in which the argument of recursion has a constant value, since otherwise the requirement of eliminability is violated. Such considerations are of little consequence, however, since any recursive definition can be turned into a direct one by purely logical methods. Cf. Carnap, *Logische Syntax der Sprache* (Vienna, 1934), pp. 23, 79.

ment; even ' $\tan \pi = \frac{\sin \pi}{\cos \pi}$ ', is a definitional transformation of an antecedent self-identity, rather than a spontaneous consequence of the definition.

What is loosely called a logical consequence of definitions is therefore more exactly describable as a logical truth definitionally abbreviated: a statement which becomes a truth of logic when definienda are replaced by definientia. In this sense ' $\tan \pi = \frac{\sin \pi}{\cos \pi}$ ', is a logical consequence of the contextual definition of the tangent. 'The altitude of Kibo exceeds six kilometers' is not *ipso facto* a logical consequence of the given definition of 'kilometer'; on the other hand it would be a logical consequence of a quite suitable but unlikely definition introducing 'Kibo' as an abbreviation of the phrase 'the totality of such African terrain as exceeds six kilometers in altitude', for under this definition the statement in question is an abbreviation of a truth of logic, *viz.* 'The altitude of the totality of such African terrain as exceeds six kilometers in altitude exceeds six kilometers.'

Whatever may be agreed upon as the exact scope of logic, we may expect definitional abbreviations of logical truths to be reckoned as logical rather than extra-logical truths. This being the case, the preceding conclusion shows logical consequences of definitions to be themselves truths of logic. To claim that mathematical truths are conventional in the sense of following logically from definitions is therefore to claim that mathematics is part of logic. The latter claim does not represent an arbitrary extension of the term 'logic' to include mathematics; agreement as to what belongs to logic and what belongs to mathematics is supposed at the outset, and it is then claimed that definitions of mathematical expressions can so be framed on the basis of logical ones that all mathematical truths become abbreviations of logical ones.

Although signs introduced by definition are formally arbitrary, more than such arbitrary notational convention is

involved in questions of definability; otherwise any expression might be said to be definable on the basis of any expressions whatever. When we speak of definability, or of finding a definition for a given sign, we have in mind some traditional usage of the sign antecedent to the definition in question. To be satisfactory in this sense a definition of the sign not only must fulfill the formal requirement of unambiguous eliminability, but must also conform to the traditional usage in question. For such conformity it is necessary and sufficient that every context of the sign which was true and every context which was false under traditional usage be construed by the definition as an abbreviation of some other statement which is correspondingly true or false under the established meanings of its signs. Thus when definitions of mathematical expressions on the basis of logical ones are said to have been framed, what is meant is that definitions have been set up whereby every statement which so involves those mathematical expressions as to be recognized traditionally as true, or as false, is construed as an abbreviation of another correspondingly true or false statement which lacks those mathematical expressions and exhibits only logical expressions in their stead.<sup>3</sup>

An expression will be said to occur *vacuously* in a given statement if its replacement therein by any and every other grammatically admissible expression leaves the truth or falsehood of the statement unchanged. Thus for any statement containing some expressions vacuously there is a class of statements, describable as *vacuous variants* of the given statement, which are like it in point of truth or falsehood, like it also in point of a certain skeleton of symbolic make-up, but diverse in exhibiting all grammatically possible variations upon the vacuous constituents of the given statements. An

<sup>3</sup> Note that an expression is said to be defined, in terms *e.g.* of logic, not only when it is a single sign whose elimination from a context in favor of logical expressions is accomplished by a single application of one definition, but also when it is a complex expression whose elimination calls for successive application of many definitions.

expression will be said to occur *essentially* in a statement if it occurs in all the vacuous variants of the statement, *i.e.* if it forms part of the aforementioned skeleton. (Note that though an expression occur non-vacuously in a statement it may fail of essential occurrence because some of its parts occur vacuously in the statement.)

Now let  $S$  be a truth, let the expressions  $E_i$  occur vacuously in  $S$ , and let the statements  $S_j$  be the vacuous variants of  $S$ . Thus the  $S_j$  will likewise be true. On the sole basis of the expressions belonging to a certain class  $\alpha$ , let us frame a definition for one of the expressions  $F$  occurring in  $S$  outside the  $E_i$ .  $S$  and the  $S_i$  thereby become abbreviations of certain statements  $S'$  and  $S'_i$  which exhibit only members of  $\alpha$  instead of those occurrences of  $F$ , but which remain so related that the  $S'_i$  are all the results of replacing the  $E_i$  in  $S'$  by any other grammatically admissible expressions. Now since our definition of  $F$  is supposed to conform to usage,  $S'$  and the  $S'_i$  will, like  $S$  and the  $S_i$ , be uniformly true; hence the  $S'_i$  will be vacuous variants of  $S'$ , and the occurrences of the  $E_i$  in  $S'$  will be vacuous. The definition thus makes  $S$  an abbreviation of a truth  $S'$  which, like  $S$ , involves the  $E_i$  vacuously, but which differs from  $S$  in exhibiting only members of  $\alpha$  instead of the occurrences of  $F$  outside the  $E_i$ . Now it is obvious that an expression cannot occur essentially in a statement if it occurs only within expressions which occur vacuously in the statement; consequently  $F$ , occurring in  $S'$  as it does only within the  $E_i$  if at all, does not occur essentially in  $S'$ ; members of  $\alpha$  occur essentially in its stead. Thus if we take  $F$  as any non-member of  $\alpha$  occurring essentially in  $S$ , and repeat the above reasoning for each such expression, we see that, through definitions of all such expressions in terms of members of  $\alpha$ ,  $S$  becomes an abbreviation of a truth  $S''$  involving only members of  $\alpha$  essentially.

Thus if in particular we take  $\alpha$  as the class of all logical expressions, the above tells us that if logical definitions be framed for all non-logical expressions occurring essentially in

the true statement  $S$ ,  $S$  becomes an abbreviation of a truth  $S''$  involving only logical expressions essentially. But if  $S''$  involves only logical expressions essentially, and hence remains true when everything except that skeleton of logical expressions is changed in all grammatically possible ways, then  $S''$  depends for its truth upon those logical constituents alone, and is thus a truth of logic. It is therefore established that if all non-logical expressions occurring essentially in a true statement  $S$  be given definitions on the basis solely of logic, then  $S$  becomes an abbreviation of a truth  $S''$  of logic. In particular, then, if all mathematical expressions be defined in terms of logic, all truths involving only mathematical and logical expressions essentially become definitional abbreviations of truths of logic.

Now a mathematical truth, *e.g.* 'Smith's age plus Brown's equals Brown's age plus Smith's,' may contain non-logical, non-mathematical expressions. Still any such mathematical truth, or another whereof it is a definitional abbreviation, will consist of a skeleton of mathematical or logical expressions filled in with non-logical, non-mathematical expressions all of which occur vacuously. Thus every mathematical truth either is a truth in which only mathematical and logical expressions occur essentially, or is a definitional abbreviation of such a truth. Hence, granted definitions of all mathematical expressions in terms of logic, the preceding conclusion shows that all mathematical truths become definitional abbreviations of truths of logic—therefore truths of logic in turn. For the thesis that mathematics is logic it is thus sufficient that all mathematical notation be defined on the basis of logical notation.

If on the other hand some mathematical expressions resist definition on the basis of logical ones, then every mathematical truth containing such recalcitrant expressions must contain them only inessentially, or be a definitional abbreviation of a truth containing such expressions only inessentially, if all mathematics is to be logic: for though a logical truth, *e.g.* the

above one about Africa, may involve non-logical expressions, it or some other logical truth whereof it is an abbreviation must involve only logical expressions essentially. It is of this alternative that those<sup>4</sup> avail themselves who regard mathematical truths, insofar as they depend upon non-logical notions, as elliptical for hypothetical statements containing as tacit hypotheses all the postulates of the branch of mathematics in question. Thus, suppose the geometrical terms 'sphere' and 'includes' to be undefined on the basis of logical expressions, and suppose all further geometrical expressions defined on the basis of logical expressions together with 'sphere' and 'includes', as with Huntington.<sup>5</sup> Let Huntington's postulates for (Euclidean) geometry, and all the theorems, be expanded by thoroughgoing replacement of definienda by definienda, so that they come to contain only logical expressions and 'sphere' and 'includes', and let the conjunction of the thus expanded postulates be represented as 'Hunt (sphere, includes).' Then, where ' $\Phi$  (sphere, includes)' is any of the theorems, similarly expanded into primitive terms, the point of view under consideration is that ' $\Phi$  (sphere, includes),' insofar as it is conceived as a mathematical truth, is to be construed as an ellipsis for 'If Hunt (sphere, includes) then  $\Phi$  (sphere, includes).' Since ' $\Phi$  (sphere, includes)' is a logical consequence of Huntington's postulates, the above hypothetical statement is a truth of logic; it involves the expressions 'sphere' and 'includes' inessentially, in fact vacuously, since the logical deducibility of the theorems from the postulates is independent of the meanings of 'sphere' and 'includes' and survives the replacement of those expressions by any other grammatically admissible expressions whatever. Since, granted the fitness of Huntington's postulates, all and only those geometrical statements are truths of geometry which are logical consequences in this fashion of 'Hunt (sphere, includes),' all

<sup>4</sup> E.g. Russell, *op. cit.*, pp. 429-430; Behmann, "Sind die mathematischen Urteile analytisch oder synthetisch?," *Erkenntnis* 4 (1934), pp. 8-10.

<sup>5</sup> "A Set of Postulates for Abstract Geometry," *Mathematische Annalen* 73 (1913), pp. 522-559.

geometry becomes logic when interpreted in the above manner as a conventional ellipsis for a body of hypothetical statements.

But if, as a truth of mathematics, ' $\Phi$  (sphere, includes)' is short for 'If Hunt (sphere, includes) then  $\Phi$  (sphere, includes),' still there remains, as part of this expanded statement, the original statement ' $\Phi$  (sphere, includes)'; this remains as a presumably true statement within some body of doctrine, say for the moment "non-mathematical geometry", even if the title of mathematical truth be restricted to the entire hypothetical statement in question. The body of all such hypothetical statements, describable as the "theory of deduction of non-mathematical geometry", is of course a part of logic; but the same is true of any "theory of deduction of sociology", "theory of deduction of Greek mythology", etc., which we might construct in parallel fashion with the aid of any set of postulates suited to sociology or to Greek mythology. The point of view toward geometry which is under consideration thus reduces merely to an exclusion of geometry from mathematics, a relegation of geometry to the status of sociology or Greek mythology; the labelling of the "theory of deduction of non-mathematical geometry" as "mathematical geometry" is a verbal *tour de force* which is equally applicable in the case of sociology or Greek mythology. To incorporate mathematics into logic by regarding all recalcitrant mathematical truths as elliptical hypothetical statements is thus in effect merely to restrict the term 'mathematics' to exclude those recalcitrant branches. But we are not interested in renaming. Those disciplines, geometry and the rest, which have traditionally been grouped under mathematics are the objects of the present discussion, and it is with the doctrine that mathematics in this sense is logic that we are here concerned.<sup>6</sup>

Discarding this alternative and returning, then, we see that

<sup>6</sup> Obviously the foregoing discussion has no bearing upon postulate method as such, nor upon Huntington's work.

If some mathematical expressions resist definition on the basis of logical ones, mathematics will reduce to logic only if, under a literal reading and without the gratuitous annexation of hypotheses, every mathematical truth contains (or is an abbreviation of one which contains) such recalcitrant expressions only inessentially if at all. But a mathematical expression sufficiently troublesome to have resisted trivial contextual definition in terms of logic can hardly be expected to occur thus idly in all its mathematical contexts. It would thus appear that for the tenability of the thesis that mathematics is logic it is not only sufficient but also necessary that all mathematical expressions be capable of definition on the basis solely of logical ones.

Though in framing logical definitions of mathematical expressions the ultimate objective be to make all mathematical truths logical truths, attention is not to be confined to mathematical and logical truths in testing the conformity of the definitions to usage. Mathematical expressions belong to the general language, and they are to be so defined that all statements containing them, whether mathematical truths, historical truths, or falsehoods under traditional usage, come to be construed as abbreviations of other statements which are correspondingly true or false. The definition introducing 'plus' must be such that the mathematical truth 'Smith's age plus Brown's equals Brown's age plus Smith's' becomes an abbreviation of a logical truth, as observed earlier; but it must also be such that 'Smith's age plus Brown's age equals Jones' age' becomes an abbreviation of a statement which is empirically true or false in conformity with the county records and the traditional usage of 'plus'. A definition which fails in this latter respect is no less Pickwickian than one which fails in the former; in either case nothing is achieved beyond the transient pleasure of a verbal recreation.

But for these considerations, contextual definitions of any mathematical expressions whatever could be framed immediately in purely logical terms, on the basis of any set of pos-

ulates adequate to the branch of mathematics in question. Thus, consider again Huntington's systematization of geometry. It was remarked that, granted the fitness of Huntington's postulates, a statement will be a truth of geometry if and only if it is logically deducible from 'Hunt (sphere, includes)' without regard to the meanings of 'sphere' and 'includes'. Thus ' $\Phi$  (sphere, includes)' will be a truth of geometry if and only if the following is a truth of logic: 'If  $\alpha$  is any class and  $R$  any relation such that Hunt ( $\alpha$ ,  $R$ ), then  $\Phi$  ( $\alpha$ ,  $R$ ).' For 'sphere' and 'includes' we might then adopt the following contextual definition: Where '---' is any statement containing ' $\alpha$ ' or ' $R$ ' or both, let the statement 'If  $\alpha$  is any class and  $R$  any relation such that Hunt ( $\alpha$ ,  $R$ ), then ---' be abbreviated as that expression which is got from '---' by putting 'sphere' for ' $\alpha$ ' and 'includes' for ' $R$ ' throughout. (In the case of a compound statement involving 'sphere' and 'includes', this definition does not specify whether it is the entire statement or each of its constituent statements that is to be accounted as shorthand in the described fashion; but this ambiguity can be eliminated by stipulating that the convention apply only to whole contexts.) 'Sphere' and 'includes' thus receive contextual definition in terms exclusively of logic, for any statement containing one or both of those expressions is construed by the definition as an abbreviation of a statement containing only logical expressions (plus whatever expressions the original statement may have contained other than 'sphere' and 'includes'). The definition satisfies past usage of 'sphere' and 'includes' to the extent of verifying all truths and falsifying all falsehoods of geometry; all those statements of geometry which are true, and only those, become abbreviations of truths of logic.

The same procedure could be followed in any other branch of mathematics, with the help of a satisfactory set of postulates for the branch. Thus nothing further would appear to be wanting for the thesis that mathematics is logic. And the royal road runs beyond that thesis, for the described

method of logicizing a mathematical discipline can be applied likewise to any non-mathematical theory. But the whole procedure rests on failure to conform the definitions to usage; what is logicized is not the intended subject-matter. It is readily seen *e.g.* that the suggested contextual definition of 'sphere' and 'includes', though transforming purely geometrical truths and falsehoods respectively into logical truths and falsehoods, transforms certain empirical truths into falsehoods and *vice versa*. Consider *e.g.* the true statement 'A baseball is roughly a sphere,' more rigorously 'The whole of a baseball, except for a certain very thin, irregular peripheral layer, constitutes a sphere.' According to the contextual definition, this statement is an abbreviation for the following: 'If  $\alpha$  is any class and  $R$  any relation such that Hunt ( $\alpha, R$ ), then the whole of a baseball, except for a thin peripheral layer, constitutes an [a member of]  $\alpha$ .' This tells us that the whole of a baseball, except for a thin peripheral layer, belongs to every class  $\alpha$  for which a relation  $R$  can be found such that Huntington's postulates are true of  $\alpha$  and  $R$ . Now it happens that 'Hunt ( $\alpha$ , includes)' is true not only when  $\alpha$  is taken as the class of all spheres, but also when  $\alpha$  is restricted to the class of spheres a foot or more in diameter;<sup>7</sup> yet the whole of a baseball, except for a thin peripheral layer, can hardly be said to constitute a sphere a foot or more in diameter. The statement is therefore false, whereas the preceding statement, supposedly an abbreviation of this one, was true under ordinary usage of words. The thus logicized rendering of any other discipline can be shown in analogous fashion to yield the sort of discrepancy observed just now for geometry, provided only that the postulates of the discipline admit, like those of geometry, of alternative applications; and such multiple applicability is to be expected of any postulate set.<sup>8</sup>

<sup>7</sup> Cf. Huntington, *op. cit.*, p. 540.

<sup>8</sup> Note that a postulate set is superfluous if it *demonstrably* admits of one and only one application: for it then embodies an adequate defining property for each of its constituent primitive terms. Cf. Tarski, "Einige methodologische Untersuchungen über die Definierbarkeit der Begriffe," *Erkenntnis* 5 (1934), p. 85 (Satz 2).

Definition of mathematical notions on the basis of logical ones is thus a more arduous undertaking than would appear from a consideration solely of the truths and falsehoods of pure mathematics. Viewed *in vacuo*, mathematics is trivially reducible to logic through erection of postulate systems into contextual definitions; but "cette science n'a pas uniquement pour objet de contempler éternellement son propre nombril."<sup>9</sup> When mathematics is recognized as capable of use, and as forming an integral part of general language, the definition of mathematical notions in terms of logic becomes a task whose completion, if theoretically possible at all, calls for mathematical genius of a high order. It was primarily to this task that Whitehead and Russell addressed themselves in their *Principia Mathematica*. They adopt a meager logical language as primitive, and on its basis alone they undertake to endow mathematical expressions with definitions which conform to usage in the full sense described above: definitions which not only reduce mathematical truths and falsehoods to logical ones, but reduce *all* statements, containing the mathematical expressions in question, to equivalent statements involving logical expressions instead of the mathematical ones. Within *Principia* the program has been advanced to such a point as to suggest that no fundamental difficulties stand in the way of completing the process. The foundations of arithmetic are developed in *Principia*, and therewith those branches of mathematics are accommodated which, like analysis and theory of number, spring from arithmetic. Abstract algebra proceeds readily from the relation theory of *Principia*. Only geometry remains untouched, and this field can be brought into line simply by identifying  $n$ -dimensional figures with those  $n$ -adic arithmetical relations ("equations in  $n$  variables") with which they are correlated through analytic geometry.<sup>10</sup> Some question Whitehead and Russell's reduc-

<sup>9</sup> Poincaré, *Science et Méthode* (Paris, 1908), p. 199.

<sup>10</sup> Cf. Study, *Die realistische Weltansicht und die Lehre vom Raume* (Brunswick, 1914), pp. 86-92.

tion of mathematics to logic,<sup>11</sup> on grounds for whose exposition and criticism there is not space; the thesis that all mathematics reduces to logic is, however, substantiated by *Principia* to a degree satisfactory to most of us. There is no need here to adopt a final stand in the matter.

If for the moment we grant that all mathematics is thus definitionally constructible from logic, then mathematics becomes true by convention in a relative sense: mathematical truths become conventional transcriptions of logical truths. Perhaps this is all that many of us mean to assert when we assert that mathematics is true by convention; at least, an *analytic* statement is commonly explained merely as one which proceeds from logic and definitions, or as one which, on replacement of definienda by definientia, becomes a truth of logic.<sup>12</sup> But in strictness we cannot regard mathematics as true purely by convention unless all those logical principles to which mathematics is supposed to reduce are likewise true by convention. And the doctrine that mathematics is *analytic* accomplishes a less fundamental simplification for philosophy than would at first appear, if it asserts only that mathematics is a conventional transcription of logic and not that logic is convention in turn: for if in the end we are to countenance any *a priori* principles at all which are independent of convention, we should not scruple to admit a few more, nor attribute crucial importance to conventions which serve only to diminish the number of such principles by reducing some to others.

But if we are to construe logic also as true by convention, we must rest logic ultimately upon some manner of convention other than definition: for it was noted earlier that definitions are available only for transforming truths, not for

<sup>11</sup> Cf. e.g. Dubislav, "Ueber das Verhältnis der Logik zur Mathematik," *Annalen der Philosophie* 5 (1925), pp. 193-208; Hilbert, *Die Grundlagen der Mathematik* (Leipzig, 1928), pp. 12, 21.

<sup>12</sup> Cf. Frege, *Grundlagen der Arithmetik* (Breslau, 1884), p. 4; Behmann, *op. cit.*, p. 5. Carnap, *op. cit.*, uses the term in essentially the same sense but subject to more subtle and rigorous treatment.

founding them. The same applies to any truths of mathematics which, contrary to the supposition of a moment ago, may resist definitional reduction to logic; if such truths are to proceed from convention, without merely being reduced to antecedent truths, they must proceed from conventions other than definitions. Such a second sort of convention, generating truths rather than merely transforming them, has long been recognized in the use of postulates.<sup>13</sup> Application of this method to logic will occupy the next section; customary ways of rendering postulates and rules of inference will be departed from, however, in favor of giving the whole scheme the explicit form of linguistic convention.

## II

LET us suppose an approximate maximum of definition to have been accomplished for logic, so that we are left with about as meager as possible an array of primitive notational devices. There are indefinitely many ways of framing the definitions, all conforming to the same usage of the expressions in question; apart from the objective of defining much in terms of little, choice among these ways is guided by convenience or chance. Different choices involve different sets of primitives. Let us suppose our procedure to be such as to reckon among the primitive devices the *not*-idiom, the *if*-idiom ('If . . . then . . . '), the *every*-idiom ('No matter what  $x$  may be, --- $x$ ---'), and one or two more as required. On the basis of this much, then, all further logical notation is to be supposed defined; all statements involving any further logical notation become construed as abbreviations of statements whose logical constituents are limited to those primitives.

'Or', as a connective joining statements to form new state-

<sup>13</sup> The function of postulates as conventions seems to have been first recognized by Gergonne, "Essai sur la théorie des définitions," *Annales des mathématiques pures et appliquées* (1819). His designation of them as "implicit definitions", which has had some following in the literature, is avoided here.

ments, is amenable to the following contextual definition in terms of the *not*-idiom and the *if*-idiom: A pair of statements with 'or' between is an abbreviation of the statement made up successively of these ingredients: first, 'If'; second, the first statement of the pair, with 'not' inserted to govern the main verb (or, with 'it is false that' prefixed); third, 'then'; fourth, the second statement of the pair. The convention becomes clearer if we use the prefix ' $\sim$ ' as an artificial notation for denial, thus writing ' $\sim$  ice is hot' instead of 'Ice is not hot' or 'It is false that ice is hot.' Where '---' and '—' are any statements, our definition then introduces '--- or —' as an abbreviation of 'If  $\sim$ --- then —.' Again 'and', as a connective joining statements, can be defined contextually by construing '--- and —' as an abbreviation for ' $\sim$  if --- then  $\sim$ —.' Every such idiom is what is known as a *truth-function*, and is characterized by the fact that the truth or falsehood of the complex statement which it generates is uniquely determined by the truth or falsehood of the several statements which it combines. All truth-functions are known to be constructible in terms of the *not*- and *if*-idioms as in the above examples.<sup>14</sup> On the basis of the truth-functions, then, together with our further primitives—the *every*-idiom *et al.*—all further logical devices are supposed defined.

A word may, through historical or other accidents, evoke a train of ideas bearing no relevance to the truth or falsehood of its context; in point of *meaning*, however, as distinct from connotation, a word may be said to be determined to whatever extent the truth or falsehood of its contexts is determined. Such determination of truth or falsehood may be outright, and to that extent the meaning of the word is absolutely

<sup>14</sup> Sheffer ("A Set of Five Independent Postulates for Boolean Algebras," *Trans. Amer. Math. Soc.* 14 (1913), pp. 481-488) has shown ways of constructing these two, in turn, in terms of one; strictly, therefore, such a one should supplant the two in our ostensibly minimal set of logical primitives. Exposition will be facilitated, however, by retaining the redundancy.

determined; or it may be relative to the truth or falsehood of statements containing other words, and to that extent the meaning of the word is determined relatively to those other words. A definition endows a word with complete determinacy of meaning relative to other words. But the alternative is open to us, on introducing a new word, of determining its meaning *absolutely* to whatever extent we like by specifying contexts which are to be true and contexts which are to be false. In fact, we need specify only the former: for falsehood may be regarded as a derivative property depending on the word '¬', in such wise that falsehood of '---' means simply truth of '¬---.' Since all contexts of our new word are meaningless to begin with, neither true nor false, we are free to run through the list of such contexts and pick out as true such ones as we like; those selected become true by fiat, by linguistic convention. For those who would question them we have always the same answer, 'You use the word differently.' The reader may protest that our arbitrary selection of contexts as true is subject to restrictions imposed by the requirement of *consistency*—e.g. that we must not select both '---' and '¬---'; but this consideration, which will receive a clearer status a few pages hence, will be passed over for the moment.

Now suppose in particular that we abstract from existing usage of the locutions 'if-then', 'not' (or '¬'), and the rest of our logical primitives, so that for the time being these become meaningless marks, and the erstwhile statements containing them lose their status as statements and become likewise meaningless, neither true nor false; and suppose we run through all those erstwhile statements, or as many of them as we like, segregating various of them arbitrarily as true. To whatever extent we carry this process, we to that extent determine meaning for the initially meaningless marks 'if', 'then', '¬', and the rest. Such contexts as we render true are true by convention.

We saw earlier that if all expressions occurring essentially

in a true statement  $S$  and not belonging to a class  $\alpha$  are given definitions in terms solely of members of  $\alpha$ , then  $S$  becomes a definitional abbreviation of a truth  $S''$  involving only members of  $\alpha$  essentially. Now let  $\alpha$  comprise just our logical primitives, and let  $S$  be a statement which, under ordinary usage, is true and involves only logical expressions essentially. Since all logical expressions other than the primitives are defined in terms of the primitives, it then follows that  $S$  is an abbreviation of a truth  $S''$  involving only the primitives essentially. But if one statement  $S$  is a definitional abbreviation of another  $S''$ , the truth of  $S$  proceeds wholly from linguistic convention if the truth of  $S''$  does so. Hence if, in the above process of arbitrarily segregating statements as true by way of endowing our logical primitives with meaning, *we assign truth to those statements which, according to ordinary usage, are true and involve only our primitives essentially*, then not only will the latter statements be true by convention, but so will all statements which are true under ordinary usage and involve only logical expressions essentially. Since, as remarked earlier, every logical truth involves (or is an abbreviation of another which involves) only logical expressions essentially, the described scheme of assigning truth makes all logic true by convention.

Not only does such assignment of truth suffice to make all those statements true by convention which are true under ordinary usage and involve only logical expressions essentially, but it serves also to make all those statements false by convention which are false under ordinary usage and involve only logical expressions essentially. This follows from our explanation of the falsehood of '---' as the truth of ' $\sim$ ---', since '---' will be false under ordinary usage if and only if ' $\sim$ ---' is true under ordinary usage. The described assignment of truth thus goes far toward fixing all logical expressions in point of meaning, and fixing them in conformity with usage. Still many statements containing logical expressions remain unaffected by the described assignments: all

those statements which, from the standpoint of ordinary usage, involve some non-logical expressions essentially. There is hence room for supplementary conventions of one sort or another, over and above the described truth-assignments, by way of completely fixing the meanings of our primitives — and fixing them, it is to be hoped, in conformity with ordinary usage. Such supplementation need not concern us now; the described truth-assignments provide partial determinations which, as far as they go, conform to usage, and which go far enough to make all logic true by convention.

But we must not be deceived by schematism. It would appear that we sit down to a list of expressions and check off as arbitrarily true all those which, under ordinary usage, are true statements involving only our logical primitives essentially; but this picture wanes when we reflect that the number of such statements is infinite. If the convention whereby those statements are singled out as true is to be formulated in finite terms, we must avail ourselves of conditions finite in length which determine infinite classes of expressions.<sup>15</sup>

Such conditions are ready at hand. One, determining an infinite class of expressions all of which, under ordinary usage, are true statements involving only our primitive *if*-idiom essentially, is the condition of being obtainable from

(1) 'If if  $p$  then  $q$  then if if  $q$  then  $r$  then if  $p$  then  $r$ ' by putting a statement for ' $p$ ', a statement for ' $q$ ', and a statement for ' $r$ '. In more customary language the form (1) would be expanded, for clarity, in some such fashion as this: 'If it is the case that if  $p$  then  $q$ , then, if it is the case further that if  $q$  then  $r$ , then, if  $p$ ,  $r$ .' The form (1) is thus seen to be the principle of the syllogism. Obviously it is true under ordinary usage for all substitutions of statements for ' $p$ ', ' $q$ ', and ' $r$ '; hence such results of substitution are, under ordinary

<sup>15</sup> Such a condition is all that constitutes a *formal system*. Usually we assign such meanings to the signs as to construe the expressions of the class as statements, specifically true statements, theorems; but this is neither intrinsic to the system nor necessary in all cases for a useful application of the system.

usage, true statements involving only the *if*-idiom essentially. One infinite part of our program of assigning truth to all expressions which, under ordinary usage, are true statements involving only our logical primitives essentially, is thus accomplished by the following convention:

(I) Let all results of putting a statement for '*p*', a statement for '*q*', and a statement for '*r*' in (1) be true.

Another infinite part of the program is disposed of by adding this convention:

(II) Let any expression be true which yields a truth when put for '*q*' in the result of putting a truth for '*p*' in 'If *p* then *q*.'

Given truths '---' and 'If --- then —,' (II) yields the truth of '—.' That (II) conforms to usage, *i.e.* that from statements which are true under ordinary usage (II) leads only to statements which are likewise true under ordinary usage, is seen from the fact that under ordinary usage a statement '—' is always true if statements '---' and 'If --- then —' are true. Given all the truths yielded by (I), (II) yields another infinity of truths which, like the former, are under ordinary usage truths involving only the *if*-idiom essentially. How this comes about is seen roughly as follows. The truths yielded by (I), being of the form of (1), are complex statements of the form 'If --- then —.' The statement '---' here may in particular be of the form (1) in turn, and hence likewise be true according to (I). Then, by (II), '—' becomes true. In general '—' will not be of the form (1), hence would not have been obtainable by (I) alone. Still '—' will in every such case be a statement which, under ordinary usage, is true and involves only the *if*-idiom essentially; this follows from the observed conformity of (I) and (II) to usage, together with the fact that the above derivation of '—' demands nothing of '—' beyond proper structure in terms of 'if-then'. Now our stock of truths embraces not only those yielded by (I) alone, *i.e.* those having the form (1), but also all those thence derivable by (II) in the manner

in which '—' has just now been supposed derived.<sup>16</sup> From this increased stock we can derive yet further ones by (II), and these likewise will, under ordinary usage, be true and involve only the *if*-idiom essentially. The generation proceeds in this fashion *ad infinitum*.

When provided only with (I) as an auxiliary source of truth, (II) thus yields only truths which under ordinary usage are truths involving only the *if*-idiom essentially. When provided with further auxiliary sources of truths, however, e.g. the convention (III) which is to follow, (II) yields truths involving further locutions essentially. Indeed, the effect of (II) is not even confined to statements which, under ordinary usage, involve only logical locutions essentially; (II) also legislates regarding other statements, to the extent of specifying that no two statements '---' and 'If --- then —' can both be true unless '—' is true. But this overflow need not disturb us, since it also conforms to ordinary usage. In fact, it was remarked earlier that room remained for supplementary conventions, over and above the described truth-assignments, by way of further determining the meanings of our primitives. This overflow accomplishes just that for the *if*-idiom; it provides, with regard even to a statement 'If --- then —' which from the standpoint of ordinary usage involves non-logical expressions essentially, that the statement is not to be true if '---' is true and '—' not.

But present concern is with statements which, under ordinary usage, involve only our logical primitives essentially; by (I) and (II) we have provided for the truth of an infinite number of such statements, but by no means all. The following convention provides for the truth of another infinite set of such statements; these, in contrast to the preceding, involve not only the *if*-idiom but also the *not*-idiom essentially (under ordinary usage).

<sup>16</sup> The latter in fact comprise all and only those statements which have the form 'If if if if  $q$  then  $r$  then if  $p$  then  $r$  then  $s$  then if if  $p$  then  $q$  then  $s$ '.

- (III) Let all results of putting a statement for ' $p$ ' and a statement for ' $q$ ', in 'If  $p$  then if  $\sim p$  then  $q$ ' or 'If if  $\sim p$  then  $p$  then  $p$ ,' be true.<sup>17</sup>

Statements generated thus by substitution in 'If  $p$  then if  $\sim p$  then  $q$ ' are statements of hypothetical form in which two mutually contradictory statements occur as premisses; obviously such statements are trivially true, under ordinary usage, no matter what may figure as conclusion. Statements generated by substitution in 'If [it is the case that] if  $\sim p$  then  $p$ , then  $p$ ' are likewise true under ordinary usage, for one reason as follows: Grant the hypothesis, *viz.* that if  $\sim p$  then  $p$ ; then we must admit the conclusion, *viz.* that  $p$ , since even denying it we admit it. Thus all the results of substitution referred to in (III) are true under ordinary usage no matter what the substituted statements may be; hence such results of substitution are, under ordinary usage, true statements involving nothing essentially beyond the *if*-idiom and the *not*-idiom (' $\sim$ ').

From the infinity of truths adopted in (III), together with those already at hand from (I) and (II), infinitely more truths are generated by (II). It happens, curiously enough, that (III) adds even to our stock of statements which involve only the *if*-idiom essentially (under ordinary usage); there are truths of that description which, though lacking the *not*-idiom, are reached by (I)-(III) and not by (I) and (II). This is true *e.g.* of any instance of the principle of identity, say

- (2) 'If time is money then time is money.'

It will be instructive to derive (2) from (I)-(III), as an illustration of the general manner in which truths are generated by those conventions. (III), to begin with, directs that we adopt these statements as true:

- (3) 'If time is money then if time is not money then time is money.'

<sup>17</sup> (1) and the two formulae in (III) are Łukasiewicz's three postulates for the propositional calculus.

(4) 'If if time is not money then time is money then time is money.'

(I) directs that we adopt this as true :

(5) 'If if time is money then if time is not money then time is money then if if if time is not money then time is money then time is money then if time is money then time is money.'

(II) tells us that, in view of the truth of (5) and (3), this is true :

(6) 'If if if time is not money then time is money then time is money then if time is money then time is money.'

Finally (II) tells us that, in view of the truth of (6) and (4), (2) is true.

If a statement  $S$  is generated by (I)-(III), obviously only the structure of  $S$  in terms of 'if-then' and ' $\sim$ ' was relevant to the generation; hence all those variants  $S_i$  of  $S$  which are obtainable by any grammatically admissible substitutions upon constituents of  $S$  not containing 'if', 'then', or ' $\sim$ ', are likewise generated by (I)-(III). Now it has been observed that (I)-(III) conform to usage, *i.e.* generate only statements which are true under ordinary usage; hence  $S$  and all the  $S_i$  are uniformly true under ordinary usage, the  $S_i$  are therefore vacuous variants of  $S$ , and hence only 'if', 'then', and ' $\sim$ ' occur essentially in  $S$ . Thus (I)-(III) generate only statements which under ordinary usage are truths involving only the *if*-idiom and the *not*-idiom essentially.

It can be shown also that (I)-(III) generate *all* such statements.<sup>18</sup> Consequently (I)-(III), aided by our definitions

<sup>18</sup> The proof rests essentially upon Łukasiewicz's proof (in his *Elementy logiki matematycznej* (Warsaw, 1929)) that his three postulates for the propositional calculus, *viz.* (I) and the formulae in (III), are *complete*. Adaptation of his result to present purposes depends upon the fact, readily established, that any formula generable by his two rules of inference

of logical locutions in terms of our primitives, are adequate to the generation of all statements which under ordinary usage are truths which involve any of the so-called truth-functions but nothing else essentially: for it has been remarked that all the truth-functions are definable on the basis of the *if*-idiom and the *not*-idiom. All such truths thus become true by convention. They comprise all those statements which are instances of any of the principles of the so-called propositional calculus.

To (I)-(III) we may now add a further convention or two to cover another of our logical primitives—say the *every*-idiom. A little more in this direction, by way of providing for our remaining primitives, and the program is completed; all statements which under ordinary usage are truths involving only our logical primitives essentially become true by convention. Therewith, as observed earlier, all logic becomes true by convention. The conventions with which (I)-(III) are thus to be supplemented will be more complex than (I)-(III), and considerable space would be needed to present them. But there is no need to do so, for (I)-(III) provide adequate illustration of the method; the complete set of conventions would be an adaptation of one of various existing systematizations of general logic, in the same way in which (I)-(III) are an adaptation of a systematization of the propositional calculus.

Let us now consider the protest which the reader raised earlier, *viz.* that our freedom in assigning truth by convention is subject to restrictions imposed by the requirement of con-

(the so-called rule of substitution and a rule answering to (II)) can be generated by applying the rules in such order that all applications of the rule of substitution precede all applications of the other rule. This fact is relevant because of the manner in which the rule of substitution has been absorbed, here, into (I) and (III). The adaptation involves also two further steps, which however present no difficulty: we must make connection between Łukasiewicz's *formulae*, containing variables '*p*', '*q*', etc., and the concrete *statements* which constitute the present subject-matter; also between *completeness*, in the sense (Post's) in which Łukasiewicz uses the term, and the generability of all statements which under ordinary usage are truths involving only the *if*-idiom or the *not*-idiom essentially.

sistency.<sup>19</sup> Under the fiction, implicit in an earlier stage of our discussion, that we check off our truths one by one in an exhaustive list of expressions, consistency in the assignment of truth is nothing more than a special case of conformity to usage. If we make a mark in the margin opposite an expression '---,' and another opposite '∼---,' we sin only against the established usage of '∼' as a denial sign. Under the latter usage '---' and '∼---' are not both true; in taking them both by convention as true we merely endow the sign '∼', roughly speaking, with a meaning other than denial. Indeed, we might so conduct our assignments of truth as to allow no sign of our language to behave analogously to the denial locution of ordinary usage; perhaps the resulting language would be inconvenient, but conventions are often inconvenient. It is only the objective of ending up with our mother tongue that dissuades us from marking both '---' and '∼---,' and this objective would dissuade us also from marking 'It is always cold on Thursday.'

The requirement of consistency still retains the above status when we assign truth wholesale through general conventions such as (I)-(III). Each such convention assigns truth to an infinite sheaf of the entries in our fictive list, and in this function the conventions cannot conflict; by overlapping in their effects they reinforce one another, by not overlapping they remain indifferent to one another. If some of the conventions specified entries to which truth was *not* to be assigned, genuine conflict might be apprehended; such negative conventions, however, have not been suggested. (II) was, indeed, described earlier as specifying that 'If --- then —' is not to be true if '---' is true and '—' not; but within the framework of the conventions of truth-assignment this apparent proscription is ineffectual without antecedent proscription of '—'. Thus any inconsistency among the general conventions will be of the sort previously con-

<sup>19</sup> So e.g. Poincaré, *op. cit.*, pp. 162-163, 195-198; Schlick, *Allgemeine Erkenntnislehre* (Berlin, 1925), pp. 36, 327.

sidered, *viz.* the arbitrary adoption of both '---' and '—' as true; and the adoption of these was seen merely to impose some meaning other than denial upon the sign '—'. As theoretical restrictions upon our freedom in the conventional assignment of truth, requirements of consistency thus disappear. Preconceived usage may lead us to stack the cards, but does not enter the rules of the game.

## III

CIRCUMSCRIPTION of our logical primitives in point of meaning, through conventional assignment of truth to various of their contexts, has been seen to render all logic true by convention. Then if we grant the thesis that mathematics is logic, *i.e.* that all mathematical truths are definitional abbreviations of logical truths, it follows that mathematics is true by convention.

If on the other hand, contrary to the thesis that mathematics is logic, some mathematical expressions resist definition in terms of logical ones, we can extend the foregoing method into the domain of these recalcitrant expressions: we can circumscribe the latter through conventional assignment of truth to various of their contexts, and thus render mathematics conventionally true in the same fashion in which logic has been rendered so. Thus, suppose some mathematical expressions to resist logical definition, and suppose them to be reduced to as meager as possible a set of mathematical primitives. In terms of these and our logical primitives, then, all further mathematical devices are supposed defined; all statements containing the latter become abbreviations of statements containing by way of mathematical notation only the primitives. Here, as remarked earlier in the case of logic, there are alternative courses of definition and therewith alternative sets of primitives; but suppose our procedure to be such as to count 'sphere' and 'includes' among the mathematical primitives. So far we have a set of conventions, (I)-(III) and a few more, let us call them (IV)-(VII), which

together circumscribe our logical primitives and yield all logic. By way of circumscribing the further primitives 'sphere' and 'includes', let us now add this convention to the set:

(VIII) Let 'Hunt (sphere, includes)' be true.

Now we saw earlier that where ' $\Phi$  (sphere, includes)' is any truth of geometry, supposed expanded into primitive terms, the statement

(7) 'If Hunt (sphere, includes) then  $\Phi$  (sphere, includes)' is a truth of logic. Hence (7) is one of the expressions to which truth is assigned by the conventions (I)-(VII). Now (II) instructs us, in view of convention (VIII) and the truth of (7), to adopt ' $\Phi$  (sphere, includes)' as true. In this way each truth of geometry is seen to be present among the statements to which truth is assigned by the conventions (I)-(VII).

We have considered four ways of construing geometry. One way consisted of straightforward definition of geometrical expressions in terms of logical ones, within the direction of development represented by *Principia Mathematica*; this way, presumably, would depend upon identification of geometry with algebra through the correlations of analytic geometry, and definition of algebraic expressions on the basis of logical ones as in *Principia Mathematica*. By way of concession to those who have fault to find with certain technical points in *Principia*, this possibility was allowed to retain a tentative status. The other three ways all made use of Huntington's postulates, but are sharply to be distinguished from one another. The first was to include geometry in logic by construing geometrical truths as elliptical for hypothetical statements bearing 'Hunt (sphere, includes)' as hypothesis; this was seen to be a mere evasion, tantamount, under its verbal disguise, to the concession that geometry is not logic after all. The next procedure was to define 'sphere' and 'includes' contextually in terms of logical expressions by construing ' $\Phi$  (sphere, includes)' in every case as an abbrevia-

tion of 'If  $\alpha$  is any class and  $R$  any relation such that Hunt ( $\alpha, R$ ), then  $\Phi (\alpha, R)$ .' This definition was condemned on the grounds that it fails to yield the intended usage of the defined terms. The last procedure finally, just now presented, renders geometry true by convention without making it part of logic. Here 'Hunt (sphere, includes)' is made true by fiat, by way of conventionally delimiting the meanings of 'sphere' and 'includes'. The truths of geometry then emerge not as truths of logic, but in parallel fashion to the truths of logic.

This last method of accommodating geometry is available also for any other branch of mathematics which may resist definitional reduction to logic. In each case we merely set up a conjunction of postulates for that branch as true by fiat, as a conventional circumscription of the meanings of the constituent primitives, and all the theorems of the branch thereby become true by convention: the convention thus newly adopted together with the conventions (I)-(VII). In this way all mathematics becomes conventionally true, not by becoming a definitional transcription of logic, but by proceeding from linguistic convention in the same way as does logic.

But the method can even be carried beyond mathematics, into the so-called empirical sciences. Having framed a maximum of definitions in the latter realm, we can circumscribe as many of our "empirical" primitives as we like by adding further conventions to the set adopted for logic and mathematics; a corresponding portion of "empirical" science then becomes conventionally true in precisely the manner observed above for geometry.

The impossibility of defining any of the "empirical" expressions in terms exclusively of logical and mathematical ones may be recognized at the outset: for if any proved to be so definable, there can be no question but that it would thenceforward be recognized as belonging to pure mathematics. On the other hand vast numbers of "empirical" ex-

pressions are of course definable on the basis of logical and mathematical ones together with other "empirical" ones. Thus 'momentum' is defined as 'mass times velocity'; 'event' may be defined as 'referent of the *later*-relation', *i.e.* 'whatever is later than something'; 'instant' may be defined as 'class of events no one of which is later than any other event of the class'; 'time' may be defined as 'the class of all instants'; and so on. In these examples 'momentum' is defined on the basis of mathematical expressions together with the further expressions 'mass' and 'velocity'; 'event', 'instant', and 'time' are all defined on the basis ultimately of logical expressions together with the one further expression 'later than'.

Now suppose definition to have been performed to the utmost among such non-logical, non-mathematical expressions, so that the latter are reduced to as few "empirical" primitives as possible.<sup>20</sup> *All* statements then become abbreviations of statements containing nothing beyond the logical and mathematical primitives and these "empirical" ones. Here, as before, there are alternatives of definition and therewith alternative sets of primitives; but suppose our primitives to be such as to include 'later than', and consider the totality of those statements which under ordinary usage are truths involving only 'later than' and mathematical or logical expressions essentially. Examples of such statements are 'Nothing is later than itself'; 'If Pompey died later than Brutus and Brutus died later than Caesar then Pompey died later than Caesar.' All such statements will be either very general principles, like the first example, or else instances of such principles, like the second example. Now it is a simple matter to frame a small set of general statements from which

<sup>20</sup>In *Der Logische Aufbau der Welt* (Berlin, 1928) Carnap has pursued this program with such amazing success as to provide grounds for expecting all the expressions to be definable ultimately in terms of logic and mathematics plus just one "empirical" primitive, representing a certain dyadic relation described as *recollection of resemblance*. But for the present cursory considerations no such spectacular reducibility need be presupposed.

all and only the statements under consideration can be derived by means of logic and mathematics. The conjunction of these few general statements can then be adopted as true by fiat, as 'Hunt (sphere, includes)' was adopted in (VIII); their adoption is a conventional circumscription of the meaning of the primitive 'later than'. Adoption of this convention renders all those statements conventionally true which under ordinary usage are truths essentially involving any logical or mathematical expressions, or 'later than', or any of the expressions which, like 'event', 'instant', and 'time', are defined on the basis of the foregoing, and inessentially involving anything else.

Now we can pick another of our "empirical" primitives, perhaps 'body' or 'mass' or 'energy', and repeat the process. We can continue in this fashion to any desired point, circumscribing one primitive after another by convention, and rendering conventionally true all statements which under ordinary usage are truths essentially involving only the locutions treated up to that point. If in disposing successively of our "empirical" primitives in the above fashion we take them up in an order roughly describable as leading from the general to the special, then as we progress we may expect to have to deal more and more with statements which are true under ordinary usage only with reservations, only with a probability recognized as short of certainty. But such reservations need not deter us from rendering a statement true by convention; so long as under ordinary usage the presumption is rather for than against the statement, our convention conforms to usage in verifying it. In thus elevating the statement from putative to conventional truth, we still retain the right to falsify the statement tomorrow if those events should be observed which would have occasioned its repudiation while it was still putative: for conventions are commonly revised when new observations show the revision to be convenient.

If in describing logic and mathematics as true by conven-

tion what is meant is that the primitives *can* be conventionally circumscribed in such fashion as to generate all and only the so-called truths of logic and mathematics, the characterization is empty; our last considerations show that the same might be said of any other body of doctrine as well. If on the other hand it is meant merely that the speaker adopts such conventions for those fields but not for others, the characterization is uninteresting; while if it is meant that it is a general practice to adopt such conventions explicitly for those fields but not for others, the first part of the characterization is false.

Still, there is the apparent contrast between logico-mathematical truths and others that the former are *a priori*, the latter *a posteriori*; the former have "the character of an inward necessity", in Kant's phrase, the latter do not. Viewed behavioristically and without reference to a metaphysical system, this contrast retains reality as a contrast between more and less firmly accepted statements; and it obtains antecedently to any *post facto* fashioning of conventions. There are statements which we choose to surrender last, if at all, in the course of revamping our sciences in the face of new discoveries; and among these there are some which we will not surrender at all, so basic are they to our whole conceptual scheme. Among the latter are to be counted the so-called truths of logic and mathematics, regardless of what further we may have to say of their status in the course of a subsequent sophisticated philosophy. Now since these statements are destined to be maintained independently of our observations of the world, we may as well make use here of our technique of conventional truth-assignment and thereby forestall awkward metaphysical questions as to our *a priori* insight into necessary truths. On the other hand this purpose would not motivate extension of the truth-assignment process into the realm of erstwhile contingent statements. On such grounds, then, logic and mathematics may be held to be conventional while other fields are not; it may be held

that it is philosophically important to circumscribe the logical and mathematical primitives by conventions of truth-assignment which yield all logical and mathematical truths, but that it is idle elaboration to carry the process further. Such a characterization of logic and mathematics is perhaps neither empty nor uninteresting nor false.

In the adoption of the very conventions (I)-(III) etc. whereby logic itself is set up, however, a difficulty remains to be faced. Each of these conventions is general, announcing the truth of every one of an infinity of statements conforming to a certain description; derivation of the truth of any specific statement from the general convention thus requires a logical inference, and this involves us in an infinite regress. *E.g.*, in deriving (6) from (3) and (5) on the authority of (II) we *infer*, from the general announcement (II) and the specific premiss that (3) and (5) are true statements, the conclusion that

(7) (6) is to be true.

An examination of this inference will reveal the regress. For present purposes it will be simpler to rewrite (II) thus:

(II') No matter what  $x$  may be, no matter what  $y$  may be, no matter what  $z$  may be, if  $x$  and  $z$  are true [statements] and  $z$  is the result of putting  $x$  for ' $p$ ' and  $y$  for ' $q$ ' in 'If  $p$  then  $q$ ' then  $y$  is to be true.

We are to take (II') as a premiss, then, and in addition the premiss that (3) and (5) are true. We may also grant it as known that (5) is the result of putting (3) for ' $p$ ' and (6) for ' $q$ ' in 'If  $p$  then  $q$ .' Our second premiss may thus be rendered compositely as follows:

(8) (3) and (5) are true and (5) is the result of putting (3) for ' $p$ ' and (6) for ' $q$ ' in 'If  $p$  then  $q$ .'

From these two premisses we propose to infer (7). This inference is obviously sound logic; as logic, however, it involves use of (II') and others of the conventions from which logic is supposed to spring. Let us try to perform the in-

ference on the basis of those conventions. Suppose that our convention (IV), passed over earlier, is such as to enable us to infer specific instances from statements which, like (II'), involve the *every*-idiom; *i.e.* suppose that (IV) entitles us in general to drop the prefix 'No matter what  $x$  [*or*  $y$ , *etc.*] may be' and simultaneously to introduce a concrete designation instead of ' $x$ ' [*or* ' $y$ ', *etc.*] in the sequel. By invoking (IV) three times, then, we can infer the following from (II'):

- (9) If (3) and (5) are true and (5) is the result of putting (3) for ' $p$ ' and (6) for ' $q$ ' in 'If  $p$  then  $q$ ' then (6) is to be true.

It remains to infer (7) from (8) and (9). But this is an inference of the kind for which (II') is needed; from the fact that

- (10) (8) and (9) are true and (9) is the result of putting (8) for ' $p$ ' and (7) for ' $q$ ' in 'If  $p$  then  $q$ '

we are to infer (7) with help of (II'). But the task of getting (7) from (10) and (II') is exactly analogous to our original task of getting (6) from (8) and (II'); the regress is thus under way.<sup>21</sup> (Incidentally the derivation of (9) from (II') by (IV), granted just now for the sake of argument, would encounter a similar obstacle; so also the various unanalyzed steps in the derivation of (8).)

In a word, the difficulty is that if logic is to proceed *mediately* from conventions, logic is needed for inferring logic from the conventions. Alternatively, the difficulty which appears thus as a self-presupposition of doctrine can be framed as turning upon a self-presupposition of primitives. It is supposed that the *if*-idiom, the *not*-idiom, the *every*-idiom, and so on, mean nothing to us initially, and that we adopt the conventions (I)-(VII) by way of circumscribing their meaning; and the difficulty is that communication of (I)-

<sup>21</sup> Cf. Lewis Carroll, "What the Tortoise Said to Achilles," *Mind* 4, N. S. (1895), pp. 278-280.

(VII) themselves depends upon free use of those very idioms which we are attempting to circumscribe, and can succeed only if we are already conversant with the idioms. This becomes clear as soon as (I)-(VII) are rephrased in rudimentary language, after the manner of (II').<sup>22</sup> It is important to note that this difficulty besets only the method of wholesale truth-assignment, not that of definition. It is true *e.g.* that the contextual definition of 'or' presented at the beginning of the second section was communicated with the help of logical and other expressions which cannot be expected to have been endowed with meaning at the stage where logical expressions are first being introduced. But a definition has the peculiarity of being theoretically dispensable; it introduces a scheme of abbreviation, and we are free, if we like, to forego the brevity which it affords until enough primitives have been endowed with meaning, through the method of truth-assignment or otherwise, to accommodate full exposition of the definition. On the other hand the conventions of truth-assignment cannot be thus withheld until preparations are complete, because they are needed in the preparations.

If the truth-assignments were made one by one, rather than an infinite number at a time, the above difficulty would disappear; truths of logic such as (2) would simply be asserted severally by fiat, and the problem of inferring them from more general conventions would not arise. This course was seen to be closed to us, however, by the infinitude of the truths of logic.

<sup>22</sup> Incidentally the conventions presuppose also some further locutions, *e.g.* 'true' ('a true statement'), 'the result of putting . . . for . . . in . . .', and various nouns formed by displaying expressions in quotation marks. The linguistic presuppositions can of course be reduced to a minimum by careful rephrasing; (II'), *e.g.*, can be improved to the following extent:

(II'') No matter what  $x$  may be, no matter what  $y$  may be, no matter what  $z$  may be, if  $x$  is true then if  $z$  is true then if  $z$  is the result of putting  $x$  for ' $p$ ' in the result of putting  $y$  for ' $q$ ' in 'If  $p$  then  $q$ ' then  $y$  is true.

This involves just the *every*-idiom, the *if*-idiom, 'is', and the further locutions mentioned above.

It may still be held that the conventions (I)-(VIII) etc. are *observed* from the start, and that logic and mathematics thereby become conventional. It may be held that we can adopt conventions through behavior, without first announcing them in words; and that we can return and formulate our conventions verbally afterward, if we choose, when a full language is at our disposal. It may be held that the verbal formulation of conventions is no more a prerequisite of the adoption of the conventions than the writing of a grammar is a prerequisite of speech; that explicit exposition of conventions is merely one of many important uses of a completed language. So conceived, the conventions no longer involve us in vicious regress. Inference from general conventions is no longer demanded initially, but remains to the subsequent sophisticated stage where we frame general statements of the conventions and show how various specific conventional truths, used all along, fit into the general conventions as thus formulated.

It must be conceded that this account accords well with what we actually do. We discourse without first phrasing the conventions; afterwards, in writings such as this, we formulate them to fit our behavior. On the other hand it is not clear wherein an adoption of the conventions, antecedently to their formulation, consists; such behavior is difficult to distinguish from that in which conventions are disregarded. When we first agree to understand 'Cambridge' as referring to Cambridge in England failing a suffix to the contrary, and then discourse accordingly, the rôle of linguistic convention is intelligible; but when a convention is incapable of being communicated until after its adoption, its rôle is not so clear. In dropping the attributes of deliberateness and explicitness from the notion of linguistic convention we risk depriving the latter of any explanatory force and reducing it to an idle label. We may wonder what one adds to the bare statement that the truths of logic and mathematics are *a priori*, or

to the still barer behavioristic statement that they are firmly accepted, when he characterizes them as true by convention in such a sense.

The more restricted thesis discussed in the first section, *viz.* that mathematics is a conventional transcription of logic, is far from trivial; its demonstration is a highly technical undertaking and an important one, irrespectively of what its relevance may be to fundamental principles of philosophy. It is valuable to show the reducibility of any principle to another through definition of erstwhile primitives, for every such achievement reduces the number of our presuppositions and simplifies and integrates the structure of our theories. But as to the larger thesis that mathematics and logic proceed wholly from linguistic conventions, only further clarification can assure us that this asserts anything at all.

# LOGICAL POSITIVISM AND SPECULATIVE PHILOSOPHY

BY HENRY S. LEONARD

1. The present essay, on logical positivism and speculative philosophy, aims to exhibit the former as a species of the latter, and thereby as a movement whose general characteristics conform to the fashion of the age. The essay itself might be regarded as an example of speculative philosophy in the traditional sense of that term. As such, it is perhaps not wholly inappropriate in a volume of essays chiefly concerned with speculative philosophy. But the essay presents a critique of speculation, resulting in an identification of logical positivism as the legitimate form for speculative philosophy. In virtue of the critique and the identification, the author would be obliged to deny that this essay constitutes a true part of speculative philosophy. It is, rather, only the prolegomenon to speculative philosophy.

The essay aims to make the following points: (1) That the scientific method is the only genuine method of increasing human knowledge; (2) That the speculative process is a part of the technique of the scientific method; (3) That all philosophic speculation is theoretically reducible to the presentation of a constructionalist system, and that logical positivism attempts to effect this reduction; (4) That logical positivism is no part of epistemology but is a type of speculative philosophy; and (5) That logical positivism is the sound technique in philosophy.

The essay starts with no definition of logical positivism. It may be construed as an attempt to describe what the author conceives logical positivism really to be.

2. Speculation is an attempt to formulate an exact account

of the nature of things. The traditional attitude has held that when this attempt was absolutely general, *i.e.*, when the attempt was one to formulate an account of the nature of *all* things, one's work was philosophical, and, even more specifically, metaphysical, that the sciences and even the other branches of philosophy confined their speculations to an attempt to formulate an account of the nature of only certain specific things, *e.g.*, the motion of physical bodies; or right, wrong, good, and evil.

It is usual to assume that a speculative account should employ terms of such generality that some small number would cover, in at least a general way, every actual and possible element falling within the scope of the enquiry. The consequence, in metaphysics, has been an attempt to exhibit the "necessity" of the concepts proposed, to rely upon this as a demonstration of the adequacy of the system.<sup>1</sup> Subsequently, the adequacy is illustrated by a sketchy derivation of a few of the elements from the original terms.<sup>2</sup>

The technique of the special sciences is quite a different matter. No attempt is made to lay down the necessity of the proposed basic terms. Frequently there is even no pretense that the concepts are adequate. The degree of their adequacy is a matter for determination by experiment. The auxiliary of experiment is a rigorous derivation of the elements by the use of logic and mathematics.

3. The aim of speculation is to increase human knowledge. The immediate motives are undoubtedly numerous: desire for power natural and social, for fame, native wonder and curiosity, a disturbing sense of being "lost," fear of the unknown, etc., etc. Undoubtedly those mentioned merge into and supplement one another even in the psychological motiva-

<sup>1</sup> For a self-conscious and lucid account of this point of view see the first chapter of Whitehead's *Process and Reality* (MacMillan, 1929), particularly section I, the definition of speculative philosophy and the subsequent explanatory comment. But in Whitehead this view does not exclude elements of other attitudes. See later sections of the same chapter.

<sup>2</sup> Spinoza, at least, is a striking exception, to this charge in its entirety, in that he made a persistent effort to elaborate a rigorous deductive derivation.

tions of a single individual, and there must surely be operative in some, if not in all, individuals still other forces than the five that have been mentioned. But whatever the more remote or subtler motivations, in every case the aim is toward an increase of human knowledge. And it is with this fact alone that we need here concern ourselves.

4. Furthermore, the immediate fact is that the aim is toward an increase in the knowledge of the speculating individual himself, and only secondarily toward one in human knowledge generally. However extensive the congratulations, agreements, vilifications, oppositions, or other reactions of persons to whom the speculator may have communicated his findings, it cannot be presumed that it was the object of his speculation to precipitate these results. It must be supposed that his interest was first to increase his knowledge of the world, and only afterwards to communicate.

5. Such an increase in human knowledge (*i.e.* in the knowledge of the speculating individual) may be brought about in two different ways: namely, by replacing doubt, or even complete absence of opinion, with knowledge and by replacing error with knowledge. The limits of present knowledge are not so well defined but that these two processes may be pursued simultaneously. Indeed any elaborate attempt to extend the limits of one's knowledge will probably include elements of both.

6. The preceding sections have briefly explained that the aim of speculation is to increase human knowledge. Nevertheless, speculation must not be baldly identified with the pursuit of such an aim. This aim serves only to designate a genus while speculation constitutes but a component in one of the species.

This aim may be pursued by the employment of either of two techniques. The first I shall call "pedagogical learning," the second the "scientific method." Beside these two methods, or some combination of them, there is no method of acquiring knowledge.

7. There can be no doubt that much of what we come to know is a product of the pedagogical method. This is pre-eminently true of knowledge acquired in childhood, but it remains a fact that far the larger share of what we learn even in adult life is learnt by this method and not by any application of the scientific method. Even the larger part of our knowledge of science is acquired by pedagogical learning.

Furthermore, knowledge that is available to us in this manner is for the most part uncritically absorbed. When, at last, a critical frame of mind is aroused, it results in the belief that such knowledge is valueless unless its content could be confirmed by some other method. This is not to say that such confirmation is always made before the proposed item is accepted, or that all beliefs adopted prior to the development of such an attitude of criticism are again rejected until such confirmation has actually reinstated them. So far as I know, Descartes is the only philosopher seriously to propose such a procedure.

But what it does mean is that these elements of belief are retained and new ones added only in virtue of a faith that other methods would confirm them if the other methods were to be employed. In the light of what is said in §6, the only available method of confirmation would be the scientific method.

8. If §6 is true, it must be the case that the only methods of acquiring philosophic knowledge are the pedagogic and the scientific, or some combination of these. One interesting result of this fact may be noted in the present section.

It has long been assumed that one of the important problems of philosophy is to exhibit the validity, or the grounds for the validity, of the scientific method. This problem is attacked by metaphysicians and epistemologists alike. But if the scientific method is the only irreducible method,<sup>3</sup> then the

<sup>3</sup> The pedagogical method is not irreducible, since it is supposed that every element of knowledge acquired through its use could be confirmed by an application of some other (*i.e.*, for me, the scientific) method. See above, §7.

method employed in discovering the grounds that validate it will perforce be the method whose validity or the grounds of whose validity is the point of contention. Thus, while we may not simply beg the question by positing its validity as a premise, still the argument seems, in a subtler sense, to be circular.

It must not, however, be supposed that the author believes this state of affairs would rob attempts to lay bare the grounds of validity for the scientific method of all their importance. Such attempts will be genuinely illuminating as to the nature of things. Neither does it relieve him of his responsibility as a philosopher for probing this problem himself.<sup>4</sup> Nor does he believe that the argument has any peculiar force because it is applied to the scientific method or because only one method was assumed valid. It may, in fact, be put in quite general terms, so general, even, that no particular method is specified.<sup>5</sup> It is the opinion of the author that some variant of this general argument is at the bottom of Whitehead's claim that "the hope of rationalism is not a metaphysical premise," but "the faith which forms the motive for the pursuit of all sciences alike, including metaphysics."<sup>6</sup> The situation is simply one to be noted and accepted. And the only conclusion to be drawn is that since the use of valid methods must in any event precede the exhibition of their validity, there can be no essential objection to postponing an examination of the grounds of this validity, even to a considerable extent.

What constitutes the grounds of the validity of the scientific method is a question which will not be examined in this

<sup>4</sup> The present paper is, however, not concerned with this problem.

<sup>5</sup> Let it be supposed there are only a finite number of valid methods available. Any attempt to exhibit their validity will be subtly circular. For the method used with each argument must be either the method whose validity is therein exhibited or some other one of the finite group. If in any case, the method is the one under consideration, the method is at that point circular. If at no point is this the case, then no argument is complete until the method it employs has been established. But this regression of arguments will perforce ultimately issue in one whose method is one of those previously under consideration.

<sup>6</sup> *Process and Reality*, Pt. II, Ch. 1, Sect. ii.

essay. The "hope of its validity" is the faith, not the premise, by which this essay, and all that it may serve to introduce, is motivated.

The distinguishing feature of §6, however, was not its insistence that the scientific method was *a* valid method, but that it was the *only* valid (irreducible) method. To the support for this claim, the further development of this essay is in part devoted. It takes two forms: a claim that "all sciences alike, including metaphysics," employ the scientific method, and a treatment of rationalism, deduction, intuitionism, and pragmatism as adjuncts of the scientific method.

9. The various sciences, natural, social, empirical, and pure, and even including metaphysics are not distinguished one from the other by their employment or non-employment of the scientific method, since one and all are built up only by the use of this unique instrument. What do distinguish them are their manner of using it and their ranges of subject matter. It is beyond the province of this paper to describe in detail the variation of manner possible in application of the scientific method or to classify the sciences with respect to their subject matter.<sup>7</sup> Review of a few general characteristics, with only a slight elaboration at selected points, will satisfy the demand of this enquiry.

Any application of the scientific method involves three distinct phases: formulation and determination of a "fixed base," generation of hypotheses concerning this base, and verification of these hypotheses by reference to the base. The "fixed base" is the field of fact that the science proposes to treat. The base of a natural science is clearly in some sense "empirical," that of a pure science, such as the science of number theory is less clearly so.<sup>8</sup> The base is "fixed" in the

<sup>7</sup> The sciences, furthermore, are not clearly isolated one from the other, so that their subject matters overlap and merge. Hence any classification on this basis, while sound within limits, would tend to introduce a misleading clarity.

<sup>8</sup> A discussion of this point is presented in § 12 below. Other comments bearing on it will be found throughout the remaining sections.

sense that of any identifiable entity it may theoretically be said definitely to belong to the base or definitely not to belong to it. This may be the case either in virtue of an enumeration of the members of the base or in virtue of a rule of classification. The base is not fixed in the sense that the membership has been completely passed in review or that the members all exist at any one time. For example, there are novel facts being constantly brought to light and even into existence by the prosecution of the empirical sciences.

The hypotheses generated constitute the account proposed by the science of the field of "fact" that it treats. These hypotheses await their verification via the reference they make to the fixed base and by examination of individual members of the fixed base. This is the process familiar in the case of the natural sciences as one ending in a laboratory experiment.

The sum total of hypotheses tentatively proposed by a science and now tentatively accepted as a result of their prior verification constitute the account offered by that science at that time of that base. This account is the subject of constant flux. Some hypotheses are discarded, others introduced, and even when the membership does not vary, the firmness with which the different component hypotheses are entrenched within the account is subject to fluctuation. All such change in the make up and character of the totality is what one refers to optimistically (albeit truly) as the "progress of human knowledge."

Under hypotheses are to be grouped all those things which for other purposes are classified as descriptive and explanatory generalizations, theories, laws, categorial classifications, *etc.*, *etc.*

10. Sciences are normally classified as "pure" (or "deductive") and as "empirical." In the light of the thesis that the scientific is the only method, this classification must be considered with some care. For the basis of the division is commonly supposed to be a difference of method.

The sciences classed as "pure" are primarily mathematics

and logic, and secondarily theoretical physics. By some, metaphysics would also be included. The method of these sciences is commonly said to be analytic and deductive, their character *a priori*. In these respects they are thought to differ from the empirical, or experimental, sciences.

The question to be considered in this section is not whether these sciences rest on experiments of the same sort as the undeniably experimental sciences, but whether or not their laws are established by a verificatory reference to some fixed base, however abstract that base. The conclusion will be more evident if we first digress to consider the methods of the so-called empirical sciences.

The question is one as to how the hypothesis of an experimental science is brought to the test. Sciences, and even hypotheses proper to a single science differ enormously with respect to the degree of complexity of this route of connection between hypothesis and verification. In some cases, the verification is direct and simple, since a description of the experiment that would supply a verification is an explicit part of the hypothesis. In other cases a more or less complex deductive chain of reasoning, logical or mathematical, is necessary before one can discover what sort of experiment would serve to verify the hypothesis. Thus the elaboration of deductive chains of interconnection is characteristic of the avowedly experimental sciences.

With respect to the so-called deductive sciences, certainly the day is past when anyone would seriously maintain that they present a system whose postulates are more self-evident than the theorems deduced. In fact the postulates of modern logistical and mathematical systems are in some cases not self-evident at all. The belief that we start with self-evident propositions and move to the establishment of propositions that are not self-evident is a belief belonging to the age of Descartes and Spinoza. To say just what is being done, is, however, a more difficult task.

11. There are two correlative characteristics of common

practice which, although undoubtedly helpful to the pure scientist, will produce a confusion in our thought unless we take them into account. I assume that the distinction between an abstract (uninterpreted) system and an interpretation for the system is familiar. We may speak also of an interpretative system, meaning thereby the body of propositions or propositional functions that result from supplying such an interpretation to an abstract system as will make all these propositions and functions true.<sup>9</sup> The characteristics of common practice to which a reference was already made are these: (1) Sometimes, when the only concern of a scientist is to exhibit an abstract system, he will designate elements of it by the names of relatively concrete entities. This serves to point to important analogies between the structure of the system with which he is dealing and that of the system which is capable of interpretation in terms of the concrete entities whose names he employs. Thus, for example, the mathematician speaks of "points," "lines," "surfaces," and "distance," even of an abstract "space," when what he is aiming to exhibit is an abstract system having only a certain analogy with the abstract system for "real" points, lines, surfaces, *etc.*, in short for "real" space.

(2) Conversely, the scientist whose concern is with an interpreted system will frequently elaborate his entire scheme without reference to the interpretative elements involved, that is to say, he will elaborate it wholly by means of abstract terms. But this in no way alters the fact that his concern is with the interpretative system.

The important question is, what is his concern? The simple fact is that his concern is different on different occasions. We shall endeavor briefly to describe this variety. In the course of this description we shall discover the extent to which the pure sciences are experimental.

<sup>9</sup> It is customary to claim that only propositions and not propositional functions may be "true." By a true propositional function, the author means a function of which every value is a true proposition.

12. Identification of the pure scientist's concerns may be simplified by asking what are the questions to which he seeks an answer.

Sometimes his question is, in terms of what basic considerations may a given body of knowledge be understood as systematic? It is natural to date the origin of geometry as a science from the time at which a scheme of demonstration replaced the simple statement of propositions as so many rules of thumb. Giving to geometry the form of a deductive system certainly did result in the introduction of many previously unknown propositions. But it also retained as components, with only occasional and slight modification, the rules of thumb which constituted the totality of previous geometrical knowledge.<sup>10</sup> In truth, it is the fact that these rules of thumb are retained, either as postulates or as theorems, which supplies the experimental evidence for the developed system. The question was, in terms of what basic considerations may a given body of knowledge be understood as systematic? Here is a system, its postulates present a group of basic considerations. A selection of the deduced theorems and of the postulates constitute the given body of knowledge. The fact that various components of the given body of knowledge appear in the deductive system constitutes the experimental evidence for the fact that this is a system answering to the demands made in the question.

Two points have been glossed over. One concerns the state of affairs when a developed system is adopted that requires the rejection or at least modification of certain elements in the given body of knowledge. Such a result, if the system is quite generally adopted, is hailed as a step of progress in human knowledge. An old error has been rooted out. This

<sup>10</sup> The transition from a mere set of rules of thumb to a deductive system was the work of generations. During the interval, disconnected demonstrations were discovered, new propositions included in the general body of knowledge. From the historical point of view, it was not the endeavor to achieve a single system that guided the whole development. But the idea of such a system dawned only when it practically existed as a fact to stare the Greek mathematicians in the face.

type of situation is considered further in § 14 and § 18 below.

The other point concerns the further conditions that must be satisfied by the postulates. It used to be assumed that these must be self-evident. Such a demand is not made today. For example, the general principles of mathematical physics are not self-evident, neither are those of certain branches of mathematics. Some further comment on these conditions appears below in § 15.

The fact that mathematics is in some of its reaches an experimental science as already described gains support from a consideration of the case of Geronnimo Saccheri (1667-1733), hailed as the unwitting discoverer of non-Euclidean geometry. In an effort to establish the famous parallel postulate of Euclid, he rigorously developed a pair of non-Euclidean geometries, replacing this postulate in these cases with a pair of alternatives. The evidence for Euclid lay in the fact that certain theorems of the non-Euclidean systems contradicted elements of the given body of knowledge which appeared as theorems of the Euclidean geometry.<sup>11</sup>

We have illustrated the experimental character of mathematics by references to geometry. In some ways an even more striking example is afforded in the case of number theory. This branch of mathematics is frequently developed today solely in terms of logical concepts and "nominal" definitions. But the fact that it is the theory of numbers which has been developed is shown only through the fact that the resultant theorems of the system actually exhibit the "rules of thumb" that constitute propositions in arithmetic. This evidence is experimental.

Saccheri's non-Euclidean geometries served ultimately to introduce a new conception of the function and objective of mathematics. They were thoroughly respectable abstract systems. Interest in abstract systems, even in such systems

<sup>11</sup> Saccheri characterized these theorems as nonsense, or inconceivable. Today they are regarded in the light described in the text. His error was the converse of Descartes', when the latter demanded self-evident first principles.

as were developed without any conception of what might supply a legitimate interpretation, increased. Mathematics thus became the science of possible structures. And from this time on the attempt to discern the structure of an interpreted system commonly took form as the search for an abstract system capable of bearing the desired interpretations. The elaboration of abstract systems for this purpose is a process whose success is constantly in need of verification by reference to the body of "rules of thumb" that supply the original interpretative material.

At the same time two new types of question begin to appear. One of them is a question of comparative structures: What fields of "fact" possess a common structure, or to what extent are the structures of this field and that common? The problem is approached from either of two ends. Occasionally an abstract system is already on hand, having been developed in response to prior interests; then the question is one as to what variety of fields of fact exhibit this structure. Occasionally something suggests the desirability of comparing the structures of two fields. Then the structures they exemplify must be discovered and compared. All problems of the sort described in this paragraph are empirical and stand in need of verifications of the sort previously described.

But one new type of question is definitely not empirical in the manner described. Here interest never leaves the field of the abstract. The problem, like that of Saccheri, is one simply of building up abstract systems. While interest in some one rather than another of these is likely to be the result of prior experience with other abstract systems, and this system is interesting merely because it is made to differ from the previously developed one in certain, specific ways, still this comparison with the original one is no check on the "rightness" or "wrongness" of the constructed system.

Here at last we seem to have come to some element of human knowledge that is not a product of the scientific method.

Knowledge of abstract systems, or of the fact that such and such are abstract systems, would seem to be elements of knowledge of another sort. Consideration of them will be reserved to § 15. Meanwhile three points may be disposed of, two of them quite simply in § 13, the third in § 14.

13. All that was said concerning geometry and number theory in the early pages of § 12 holds with even greater force when we consider theoretical physics. Here again we have a deductive system, but the verificatory test of its postulates and definitions lies in the fact that one is enabled therefrom to deduce the usual laws of experimental physics. Thus, as the early geometrical "rules of thumb" and the propositions in arithmetic supplied the fixed bases by reference to which the deductive systems for Euclidean geometry and number theory could be verified, so the laws of experimental science provide, at least in part, the fixed base accounted for by, and verificatory of, theoretical physics.

It was said, however, that the experimental character of physics was even more evident than that of mathematics. This is because of the fact that theoretical physics permits predictions of certain items of "brute fact" which could not be predicted so long as we restricted ourselves to those consequent theorems which are the body of laws belonging to experimental physics, and experimental check on these predictions constitutes a further source of verification. The famous Michelson-Morley experiment and the astronomical observations made during solar eclipses and applied to the verification of Einsteinian physical theory are cases in point. In the light of this, and also in view of the fact that when theoretical and experimental physics disagree (say as to the scope of a law of experimental physics), experimentation is resorted to as arbiter, it might be said that the fixed bases of theoretical and experimental physics are identical.

The second point to be considered is the state of affairs

with respect to logic. It is the only one of the so-called *a priori* sciences not yet to have been considered.<sup>12</sup>

The first thing to observe is that a large body of deductive practice was already in existence, and the distinction between valid and invalid was already operative before the science of logic took shape. The history of logic up to a date so recent even as to include the larger bulk of the history of symbolic logic, has been a progressive systematization of this practice. As such, it has been subject to verification by reference to the practice in much the same way in which geometry and number theory are verificatory. On the other hand, a certain legislative character<sup>13</sup> to the science has been more in evidence than in the other sciences so far examined.

Certain recent developments, however, such as that of non-Aristotelian logics, the development of matrix methods in logic and the discovery that every proposition in logic is a tautology, make necessary a further treatment of the subject.

So far as the mere statement of postulates and theorems belonging to a non-Aristotelian logic is concerned, we have nothing other than an abstract system, and if this were all there was to it, discussion could be postponed to § 14, where abstract systems are treated. There is, however, this difference. The elaboration of abstract mathematical systems, *i.e.* the deduction of theorems, is conducted in accordance with deductive processes describable as valid in the Aristotelian logic. In the case of non-Aristotelian logics, the very rules of procedure in accordance with which theorems are deduced are non-Aristotelian. Their reliability, or validity, is to be checked by reference to a conception of the system in terms of a finite number of possible values for variables, functions that always take some one of these same enumerated values, matrix definitions of the functions in terms of the values they possess given the values of the arguments, and specification of a "law" of the system as any function in-

<sup>12</sup> Unless one include metaphysics among the *a priori* sciences.

<sup>13</sup> See below, § 14.

variably possessing some one or more of the "designated" values.<sup>14</sup> The inspection of the appropriate matrix soon reveals the validity or non-validity of any rule of procedure.

So regarded, the non-Aristotelian system is established and its rules of procedure are shown valid by a perfect induction. The case is not one in which further experiment could show the theory wrong, because the theory is checked in every possible case: in the case of each element of the fixed base (*i.e.* of every row in the matrix).

However, interest in this very artificial base is brought about by noting that the abstract system which is interpretable as the Aristotelian logic of propositions is capable of formulation in terms of two values and matrix definitions of certain functions involved. The non-Aristotelian system is an analogous one, possessing more than two values. So regarded, the whole thing is an abstract system awaiting a useful interpretation.

14. It has been remarked before<sup>15</sup> that to a certain extent the pure sciences are "legislative." That is to say, they dictate, or lay down, a distinction between true and false which no amount of common understanding or "insight" could disturb. It is fiat, *a priori*. How does the existence of such a state of affairs comport with the claim that all knowledge is derived from application of the scientific method?

First it must be observed that a "legislative" character is not peculiar to the pure sciences, but pertains to experimental sciences as well. Biology informs us that the whale is *not* a fish no matter how much it "looks like" one. Astronomy informs us that the sun does *not* rise, no matter what the appearances and no matter how often we say that it does. Physics informs us that heat is not a material substance, no matter how much like one it behaves (in its transmission prop-

<sup>14</sup> There is usually only one "designated" value. For an illuminating discussion in English of non-Aristotelian logics and of matrices in logic, see the *Symbolic Logic* by C. I. Lewis and C. H. Langford (Century Co., 1932), ch. VII.

<sup>15</sup> See above, pp. 134. 138. Also cf. § 18, below, on the *a priori*.

erties, *etc.*). Nor is the table a placid, continuous body, appearances notwithstanding. Each of these dicta is a fiat, a pronouncement contradicting a long established and natural belief. The experiential evidence for the beliefs is certainly as good as much of that which serves to verify scientific hypotheses.

Such legislative elements in the avowedly experimental sciences offer little difficulty: "It's all," we say, "just a matter of what you mean. The sciences don't legislate anything out of existence. But what is *meant* by a 'fish' in science is something different from what is meant by a 'fish' in everyday discourse. Similarly with all the other cases. And if you attend to the *scientific* meanings of the terms you will find that experience verifies the scientist's contentions."

This account is certainly an accurate one, and illuminating as far as it goes. Only two more things need be said in connection with it. The first concerns the reason why the scientist used the name "fish" to designate his new concept. It is because the denotations of the scientific and the everyday concepts are so nearly identical. It is illuminating, rather than confusing, to use the same term. It points to the identity of the experiential facts which he is trying to understand with those compassed by the everyday concept.

But why then, and this is the second point, did he not employ a definition which is exactly equivalent in denotation? At least two reasons may be found for his decision. In the first place, the everyday concept is vague, and its denotation is correspondingly indeterminate. While there is no doubt that trout, cod, and whales are fish, and that dogs, cats and beaver are not fish, when it comes to classifying oysters, lobsters, octopi, seals, eels, and walruses, the everyday concept is too vague to afford us any clue as to whether they are or are not fish. Thus the scientific concept, which would be precise, must resort to criteria that are not considered in the everyday concept. Within the indeterminate border region the scientist is wholly at liberty to divide the species however he

may wish. That is, he may choose his definition of "fish" so as to make whatever division of the doubtful cases seems appropriate.

But this freedom of choice is seldom exercised by first making an arbitrary division and then by seeking a definition that would effect the division made. Attention is directed to the undeniable fish and the undeniable non-fish. Such further common and peculiar characteristics of the fish are sought as would suffice to effect a division of the doubtful cases. It is desirable that the characteristics selected should be logically, psychologically, and experimentally manageable. Pragmatically the best contender cuts off whales and porpoises as non-fish. Since, on a coordinate basis for classification of "land animals," these two species would be brought under the genus "mammal" and would not be left over as unclassified, the pragmatic argument in favor of the criteria is strengthened and the new definition is framed in terms of these criteria.

Thus the experimental sciences are legislative, but only in virtue of their definitional activities. The choice of definitions is a pragmatic affair, and the consequences of these pragmatic choices are certain legislative confirmations, corrections and extensions of common sense everyday usages.<sup>16</sup>

When we come to consider the pure sciences again, we find the situation with respect to their legislative character almost identical with the case just discussed. Logic, for example, tells us that " $p$  is false; therefore if  $p$ , then  $q$ ," is a valid form of argument, while, "if  $p$ , then  $q$ , and  $p$  is false, therefore  $q$  is

<sup>16</sup> My dependence on C. I. Lewis's theory of the *a priori*, as developed in his *Mind and the World-Order* (Scribners, 1930), is apparent. My treatment of common sense, or accepted usage, as a factor controlling the pragmatic choice of concepts and of definitions, is, I believe, a departure from his explicit account, although it is not inconsistent with his position. Also that this legislative character results from a combination of pragmatic definition and experimentation, and that it is common sense that is legislated to, *i.e.* whipped into conformity by the *a priori* definition and the experiment, is a departure from him. A further discussion of the rôle of common sense appears below, § 18.

false," and "All  $S$  is  $P$ ; therefore some  $S$  is  $P$ ," are not valid. Similarly, the mathematician tells us that, " $\infty + 36 = \infty$ ," and that the number of points in a straight line is as great as the number in a square erected on that line. All of these things we find more or less surprising. Some of us never can see that they are true.<sup>17</sup>

But again it is all a question of what you mean. There is the same vague, everyday concept, the same need for extended precision, the same attempt to conform within limits to the denotations of everyday concepts, and the same pragmatic motivation for employing the everyday term even while, again for pragmatic reasons, the scientific concept "corrects" the common sense. Thus once more the legislative is the definitional and pragmatic.

15. We come at last to the question as to how we know an abstract system. In earlier sections,<sup>18</sup> we have had occasion to discuss the method in which we make use of an abstract system to acquire a knowledge of a certain field of "fact." The problem of interpreting such systems is already settled as one to be approached by a use of the scientific method. Here we are concerned only with the questions, in what does the knowledge of an abstract system consist and how is this knowledge acquired?

Any enquiry as to how we know an abstract system will be simplified by first asking, what is an abstract system? Certainly, this point is controversial, but we cannot debate the issue here. A mere statement of our position must suffice. But before we can answer this question we must take notice of a defect in the usual manner of presenting abstract systems.

An abbreviative device is commonly employed in the presentation of abstract systems. While it is certainly legitimate to employ any such devices as will be an aid in the organization and acquisition of human knowledge, failure to appreciate

<sup>17</sup> Try teaching an elementary course in logic if you do not believe this!

<sup>18</sup> See above, §§ 11-13.

the fact that they are being used is likely to thwart attempts to understand what has gone on.

The accepted mathematical technique has been a presentation of this form :

A class  $K$  of elements  $a, b, c, \text{ etc.}$ , and a relation  $R$  such that :

I If  $a$  and  $b$  are distinct elements of  $K$ , then either  $aRb$  or  $bRa$ .

II If  $aRb$  is true, then  $a$  and  $b$  are distinct.

III If  $aRb$  and  $bRc$  are both true, then  $aRc$  will be true.

*Theorems:*

I. . . . . *etc.*<sup>19</sup>

When it comes to supplying an interpretation for this abstract system, we say something like this: "Let  $K$  be the class of points on a rectilinear segment and  $R$  the relation 'is to the right of.' Then all the postulates are satisfied." But what are the postulates that we are testing? They are *not* the three postulates just stated. Rather they are :

I (As stated)

II If  $a$  is a  $K$  and  $b$  is a  $K$  and  $aRb$ , then  $a$  and  $b$  are distinct.

III If  $a, b,$  and  $c$  are elements of  $K$ , then if  $aRb$  and  $bRc$  are both true,  $aRc$  will be true.

Or, if you prefer, there is only one complex postulate, having the form :

If  $a, b, c, \text{ etc.}$  are elements of  $K$ , then

I If  $a$  and  $b$  are distinct, either  $aRb$  or  $bRa$ .

II If  $aRb$ , then  $a$  and  $b$  are distinct.

III If  $aRb$  and  $bRc$  are both true, then  $aRc$  will be true.<sup>20</sup>

The psychological motive for the mathematician's practice is this. Any interpretative postulate is true for each choice of

<sup>19</sup> Adapted from E. V. Huntington's *The Continuum* (Harvard University Press, 2nd ed. 1917), p. 10.

<sup>20</sup> The form of the mathematician's thought, I believe, is more closely represented by this second formulation. For reasons not relevant to the present issue, the first formulation is *theoretically* preferable. The second has some psychological advantages. It will not affect the applicability of our conclusions if we consider only the first from now on. This, in virtue of the theorem:  $p \supset q. p \supset r. \equiv p \supset qr$ . (See Whitehead and Russell's *Principia Mathematica*, \*4.76).

$a$ ,  $b$  etc. that makes the antecedent "If  $a$  is a  $K$  and  $b$  is a  $K$ ," false. This is the case in virtue of the theorem declaring that a false proposition implies any proposition.<sup>21</sup> Thus to ascertain the truth of the postulate, it is only needful to attend to those choices of  $a$ ,  $b$ , etc., that make the antecedent true. The mathematician implicitly recognizes this situation, suppresses explicit mention of the antecedents and, by a sort of "gentlemen's agreement," focuses attention solely on those choices in which the antecedent will be true. All of this reduces the quantity and complexity of the material that must be kept in mind, and is on that account desirable.

Let us now attend to this expanded, more accurate, formulation for an abstract system. Of what does such a system consist? Briefly of this: a set of paradigmatic formulae related one to the other as postulates, definitions,<sup>22</sup> theorems and deductions. These formulae are neither true nor false, the "postulates" are not asserted, the "definitions" do not define, the "theorems" are not demonstrated. But the formulae require only an interpretation in order that they be transformed into a system of genuine propositions, true or false as the case might be. The "nominal" definitions become "real," the paradigmatic deductions become valid arguments, no matter whether the premises and conclusions derived by this interpretation be true or false.

One more point requires comment. Neither the world nor any part of it (except such parts of it as are deductive systems of formulae) is a system. When we say it is a system, we mean that all, or some important part, of our knowledge concerning it may be expressed in a system, *i.e.* in a deductively organized group of "true" formulae or propositions. This organization of propositions in a deductive scheme represents no order in the facts; there is no such thing as "therefore" in nature, but the order is in our noetic or-

<sup>21</sup> *Principia Mathematica*, \*2.21.

<sup>22</sup> It is customary to regard the definitions as no part of the system. The author takes exception to this point of view. On definition, see below, § 18.

ganization of the facts. Nature is conjunctive, not implicative.

Knowledge of an abstract system consists, first of all, in the recognition of the fact that a certain complex visible geometrical pattern (the black marks on the paper) is symbolic, is in fact the presentation of a group of paradigmatic formulae and paradigmatic deductions. Beyond this, it consists in a knowledge of the fact that the paradigm is consistent, that the paradigmatic deductions are valid, and the paradigmatic definitions legitimate.

With our recognition of the black marks as symbolic, as paradigmatic formulae, we shall not further concern ourselves.<sup>23</sup> Consistency and independence are easily disposed of. The only known technique is discovery of a multiplicity of possible and non-possible interpretations. This clearly is a verificatory procedure, and makes use of the scientific method. To determine the legitimacy of a nominal definition,<sup>24</sup> it is needful only to ascertain that the definiendum has not appeared in antecedent portions of the formulation, that it does not appear in the definiens, and that the definiens has already been established as a paradigmatic formula inherent in the system.<sup>25</sup>

That these requirements are satisfied is determined by a review of the system, hence by a verificatory experiment. The only feature remaining to be considered is our knowledge of the validity of the paradigmatic deductions.

With respect to the recognition of validity in an abstract system, it is important that we distinguish two types of prac-

<sup>23</sup> The question is, however, important, and ought ultimately to be investigated.

<sup>24</sup> The paradigmatic definition is nominal. Its correlate in the interpretation may be a real definition.

<sup>25</sup> Some legitimate types of definition would require a slight revision of these requirements. Such are, for example, the "recursive definitions" of Carnap's *Die Logische Syntax der Sprache* (1934). But the revision required would not illuminate the nature of the present problem and would only complicate the treatment of an already complex situation. The solution would remain what it is here.

tice common in the treatment of abstract systems. There is logistic, in which the "deductions" are the result of the mechanical applications of certain specified rules of procedure. And there is the type of development more usual in mathematics, where the deductions result from the direct use of commonly recognized types of valid procedure. In logistic we have the highest degree of abstraction. One demand made of proposed interpretations will be that they exhibit the rules of procedure as valid.

How do we see that the process is valid in the less abstract case? In the same way in which we see that the interpretations for the more abstract logistic result in valid processes. But how do we know that any of these are valid? The only answer is, by an intuition.<sup>26</sup>

Before leaving this section, we should observe that while, in our knowledge of the more concrete of the abstract systems, we rely immediately on an intuitive recognition of validity, and in the more abstract logistic, we rely on the direct perception of the fact that none but explicit rules of procedure have been employed, the search for an interpretation of the latter which would make the rules of procedure valid steps of deduction, is one that is carried on by application of the scientific method. Likewise, the converse problem of finding an abstract logistic which will bear the desired interpretation is solved by an application of scientific method.

16. We have now completed a survey of the natural and the pure sciences. In each case, we have found the scientific method to be the one actually employed. Parallel to the division of natural and pure science we have discovered a division in the "fixed bases" which these sciences would treat and to which they refer for verification. On the one side lay immediate sensory experience, on the other usages and intuitions.

<sup>26</sup> Intuitions of validity are considered again in connection with the more general treatment of intuition. See below, § 18.

We have not considered the social sciences, ethics, aesthetics, and metaphysics. These omissions are in part due to a desire not to extend unduly this phase of our enquiry, in part to the fact that the "proper" method in metaphysics is the point I wish to establish. The question is not one as to what method has been employed, but one as to what method ought to be employed.

It is appropriate, and reassuring, to conclude this phase of our enquiry with two quotations from Professor Whitehead's *Process and Reality*. With his account of the objective of Speculative Philosophy,<sup>27</sup> we cannot wholly agree. But intermingled with the discussion of this objective are comments on method, with many of which we are in complete agreement. Thus he says, "But the accurate expression of the final generalities is the goal of discussion, not its origin. Philosophy has been misled by the example of mathematics; and even in mathematics the statement of the ultimate logical principles is beset with difficulties, as yet insuperable."<sup>28</sup> This is to suggest that deduction is a servant of the scientific method, not an alternative. "That we fail to find in experience any elements intrinsically incapable of exhibition as examples of general theory, is the hope of rationalism. This hope is not a metaphysical premise. It is the faith which forms the motive for the pursuit of all sciences alike, including metaphysics."<sup>29</sup>

17. The object of this paper is to develop a more acute appreciation of the nature of speculative philosophy. We are now in a position to affirm that speculation constitutes one stage in the application of the scientific method. It is in fact the name for that stage in which we are engaged in the framing of hypotheses that are subsequently to be subjected to experimental verification. Thus speculation is a com-

<sup>27</sup> See his definition of Speculative Philosophy, *op. cit.*, Pt. I, Ch. 1, Sect. i. Also his subsequent explanatory discussion, Pt. I, Ch. 1, *passim*.

<sup>28</sup> *Ibid.*, Pt. I, Ch. 1, Sect. ii.

<sup>29</sup> *Ibid.*, Pt. II, Ch. 1, Sect. ii.

ponent element in the application of the only possible method for acquiring human knowledge, and both the sciences and philosophy are speculative.

Instances of speculation differ widely one from the other with respect to the continuity or discontinuity of the speculation, that is with respect to the frequency with which the speculation is interrupted by attempts at verification. These interruptions, however, may be overt, as in the case of the scientist who goes into the laboratory, or covert as in the case of the logician or philosopher who verifies hypotheses through a mnemonic review of usages and intuitive attitudes. It is customary to regard as speculative only those efforts at speculation which are relatively free from overt verificatory interruptions. Thus we speak of "theoretical," of "speculative," physics and "speculative" philosophy, in contrast with the "experimental sciences." But, for the present, at least, the similarities previously indicated are of more importance than the differences.

The distinction between speculative philosophy and the special sciences is merely one as to the scope or inclusiveness of the spheres that one endeavors to deal with by means of his scheme of hypotheses.

18. We have completed in the preceding sections the first chapter in a prolegomenon to speculative philosophy. That chapter has aimed at laying down as a first principle the unique propriety of the scientific method, both in science and in philosophy. The variety of "fixed bases" treated by different sciences (including philosophy) has been illustrated by examination of a variety of sciences. Speculation, that is, legitimate speculation, has been identified with one stage in the application of the scientific method.

To attempt adequate discussion of the problems still unmentioned relevant to a thorough elucidation of the proper sphere of philosophic speculation and the detailed form which use of the scientific method takes on when applied to philosophic problems, would launch us in a treatment fully as com-

plex as that which we have just completed. Thus no such comprehensive survey can be undertaken here. Nevertheless it will not be out of place to indicate in a concluding section the general outlines which a further development would follow.

The ultimate check on any theory lies in the capacity it gives to the holder to predict elements, and patterns of the elements of possible immediate experience. The check consists in making such predictions by use of the theory and performing those operations that would bring about the predicted experience if the theory were correct. This is in close agreement with at least half the burden of C. I. Lewis's argument in his *Mind and the World-Order*. The theory goes a step beyond what has been established in the foregoing sections in virtue of its limitation of ultimate verification to immediate, one might call it solipsistic, experience. In view of it the previous treatment of intuition will require extension.

The agent of speculation is a free imagination. Psychological characteristics of the human being impose an enormous restriction upon the freedom of the imagination. But over and above this restriction, there is required some directing principle. Consciousness of undirected freedom paralyzes the speculating agent. Some control, even if an arbitrary one, is requisite to the speculator. The principles of this control may be formulated as so many "rules of procedure." The sanction of the rules proposed is pragmatic, both in the general sense already described and in the special sense soon to be described.

These rules of procedure are wholly distinct from certain "rules of judgment." It is by application of the rules of judgment that theories are found to be right or wrong. The rules of procedure only indicate what sorts of theory it is worthwhile to concoct. Theories of the sorts indicated are some of them right, some wrong. Theories of other sorts may be right but it is hardly profitable to try to discover what these are. There is a venerable rule of procedure that

tells us, when we are dealing with any expression capable of truth or falsity, that we should *first* find out what the expression means and *then* seek to discover whether it is true or not.<sup>30</sup> Sound as it is, a contrasting procedure is sometimes<sup>31</sup> more appropriate. That is, it is sometimes proper *first* to decide what is true and then to find out what this means.

The propriety of this procedure is a corollary of certain rules of procedure that may be formulated as follows :

1. One should respect (but not be bound by) common sense.
2. One should respect (but not be bound by) the usages of "natural language" (a vehicle of common sense).
3. One should respect (but not be bound by) intuitive insights.
4. One should respect (but not be bound by) pedagogical learning.

This respect is reflected by the general acceptance of the pronouncements of these "founts of wisdom" as true, while speculation is confined to an enquiry into what might be meant by the accepted pronouncements.

The four "founts of wisdom" referred to in the four rules, may be collectively named the "social given." Then the four rules are conjunctively equivalent to the following :

One should respect (but not be bound by) the social given.

Taking the social given as a matrix of generally true propositions in no way imposes upon nature, since these propositions still await definition. The definition of them must be such as to *make* them true. An illustration of the type of speculation relevant here was given previously in our discussion of the biological definition of a "fish."<sup>32</sup>

That the respect demanded is not absolute and servile acceptance of the social given, is noted in the parenthetic clause

<sup>30</sup> The analysis of the meaning of true-false expressions generally proceeds by an analysis of the meanings of constituent concepts (expressions, either functions or arguments) which are neither true nor false.

<sup>31</sup> The value and importance of the procedure is strikingly illustrated and the process is clearly described by, e.g., Wm. James in his essay on *Pragmatism*.

<sup>32</sup> See above, § 14.

of the rules of procedure. The difference between respect and acceptance was illustrated in the extent to which biological definition resulted in a contradiction of common sense belief regarding what is, and what is not, a fish. Such modification in the interests of simplicity of the theoretical structure are desirable. The object is to remove vagueness, ambiguity, discontinuity, and inconclusiveness from the social given. But the removal must be effected by the use of an efficient, humanly manageable, scheme of ideas.

It may reasonably be asked, however, what you are "bound by." You must "respect" the social given, but are not "bound by" it. Is then the whole business an arbitrary game? Not at all. The thing that binds is always a rule of judgment, that is a test by reference to immediate experience. If we should say, arbitrarily, that element *A* of the social given is true and that the meaning of *A* is *B*, this would not suffice to make *B* true. That *B* is true is a theory which must be capable of experiential verification. If the evidence does support *B*'s truth, well and good, but if not, then we are obliged to modify our prior claims. We may either (1) revise our judgment of *A*, and claim it is false, or (2) reject *B* as a proper definiens for *A* and substitute another *B'*, or (3) do both of these things. The rule of respect for the social given would favor the second alternative, but the fact that the respect is not to be servile, leaves the other two as genuine alternatives that may be seized upon if any consideration analogous to those described in the "fish story" should suggest that *A* might better be modified. It is perhaps worth a comment that the very search for and scientific testing of *B* might be indirectly responsible for a decision to modify *A*.

What we are engaged in is an essay in speculative definition. Lewis has pointed to the definitive character of the *a priori*.<sup>33</sup> We are constructing the *a priori*. But the construction is guided by a rule of procedure demanding respect for the social given. From one point of view no experience can

<sup>33</sup> *Op. cit.*

show the construction to be wrong. It was this point of view that Lewis was constrained to establish. But from another point of view experimentation will test the construction. Does it generally validate the social given? The search for such definitions as will do just this and the willingness to reject definitions that fail in this respect is the basis for denominating this branch of intellectual enquiry speculative.

Lewis has called attention to the variety of available *a priori* schemes and has asserted that the choice between them is pragmatic. The position here developed is in entire agreement. Respect for the social given restricts the range of choice. But the rule of respect is pragmatic on two counts. First, some rule is necessary in order that the imagination can be brought under efficient control. Second, this particular rule permits adoption of a great mass of already efficient attitudes and beliefs, and in this way increases the efficiency of its product.

Finally we must consider what are the "ground elements" or basic terms (and relations) in terms of which the definitions are to be built up. It must be observed immediately that these elements need not be the epistemologist's solipsistic elements of immediate experience. All that is requisite is that if they are not, then the solipsistic elements be definable in terms of the ground elements. Indeed all elements must be so definable. This possibility of defining the solipsistic elements provides the connecting link between them and all other elements, ground or constructed, and so makes knowledge of the others through the solipsistic elements possible.

We have slipped by degrees into a use of the language of logical positivism. It is the opinion of the author that this modern movement in speculative philosophy offers the greatest promise. Unfortunately, it cannot be judged by its fruits, they are still so meager. Neither have positivists become sufficiently aware of the fact that their work is not epistemological. Logical positivism is the true modern representative of speculation in philosophy, not an annihilator of existents, but an analyzer of meanings.

# THE NATURE AND STATUS OF TIME AND PASSAGE

BY PAUL WEISS

## 1. *The Analytic Properties of Time*

WITH characteristic brevity and subtle shrewdness, St. Augustine wrote the universal commentary on all discourses on the nature and status of time. "Who is able so much as in thought to comprehend it, so as to express himself concerning it? And yet what in our usual discourse do we more familiarly and knowingly make mention of than time? And surely we understand it well enough, when we speak of it; we understand it also, when in speaking with another we hear it named. What is time then? If nobody asks me I know; but if I were desirous to explain it to one that should ask me, plainly I know not." Everyone knows what time is; the characterizations of it in folklore, proverbs, mythology, daily life and in sophisticated discussion are in such accord as to drive one almost to believe that its nature has been indelibly and clearly engraved upon the very souls of men. Yet this very spontaneous and unanimous agreement, expressed in metaphor and imagery, runs parallel with an equally spontaneous and almost unanimous agreement that all discussions of it have ended in paradox, if not in absurdity. In describing time we seem somehow to lose it, and know as surely as the rest that we have lost it. It is apparently richer than any discourse can possibly be; yet somehow adequately understood by even the least of us. There seems to be no difficulty in knowing it, but only in saying what we know. With the history of universal failure behind, and with the wisdom of St. Augustine before us, the attempt to describe the nature

<sup>1</sup>A draft of a chapter of a forthcoming book.

and status of time must nevertheless be made again. If the most that can be achieved is a more plausible and adequate account than any offered in the past, it is the least that any philosopher can attempt to do.

Time is no phantasm, private impression, form of intuition, abstraction or measure; it is integral to the real and is to be abandoned only at the price of losing the world. But the attempt to explain the place of time in the scheme of things is best deferred until its essential features have been delineated. The discordant accounts of the status of time, offered by philosophers and scientists, are discordant only because there is a modicum of agreement about the object of their discourse; otherwise they would be talking of entirely different things with only an ambiguous word in common. The time that Zeno rejects as being unreal has features which made it possible for Aristotle to say that he rejects it wrongly; the time that Newton describes as being the sensorium of God has features which make it possible for Einstein to say that it is inseparable from space. These opposing theorists are one with respect to certain characters of time; they differ with regard to others or as to the consequences to be drawn from what has already been admitted by all. The basic problem is to discover the features which all of them would admit, whether or not they face the consequences of their admissions.

The first and foremost difficulty is to be found in tearing time away from the body of things, so as to form the concept of an abstract or empty time (I use these terms interchangeably). Though time, capitalized, pictorialized and reified, is a familiar topic of daily life, art, science and philosophy, as apart from things, it is something to be grasped with difficulty and perhaps even with reluctance. But whether or not "empty time" denote something with a being of its own, or reflect merely some legitimate or illegitimate abstraction from existence, it will, I think, be agreed that its features are manifest in concrete time, and that it designates something extended, capable of division into a multitude of moments. A

“moment” is a subdivision of time. Anything that can be divided into smaller portions is, with reference to those smaller subdivisions, a “stretch of time,” so that all divisible moments are stretches. All stretches of time contain moments and all moments are contained within a stretch of time. The whole of time is a stretch which can never be a moment, and the smallest division of time is a moment which can never be a stretch. Every stretch, other than the whole of time, is also a moment, and every moment, other than the smallest, is also a stretch. From the standpoint of the whole of time, every stretch is a moment, though not every moment need be a stretch. Whatever, then, is said of the relation of the whole of time to the moments of time must also apply to the relation of the moments of time to any of their contained moments, and whatever is said of the relation of the moments of their containing stretches, must also apply to the relation of the stretches of time to the whole of time.

In order that there be time, it is not enough that there be a multitude of moments; unless the moments are in a definite order, there is no time. A single moment is a moment of time only because there is an antecedent or a predecessor; any stretch is a stretch of time only because within it there are moments in the relative order of before and after. A single moment without antecedent or predecessor, or a stretch within which no antecedent and subsequent sections could possibly be remarked, could never characterize any situation in which a past and a future were in the order of earlier to later. But a time which does not permit of the possibility of a past preceding a future is “eternity” spelt incorrectly, and deprived of all possible reference to a changing world.

The moments of time are related as before and after by a relation which has three properties—connexity, asymmetry and transitivity. The relation of the moments is connexive; i.e. any pair of them whatsoever, if they are not in the relation of stretch and moment, are such that one is before the

other. But if it be admitted that the nature of concrete time varies with a difference in the rates and kind of change entities undergo, one must acknowledge that the moment of a mental event is other than the moment of a physical event, and is neither before nor after it. That the time of the mental event is a stretch of which the time of the physical event is a moment is an hypothesis that seems never to have had the consideration it deserves. The usual supposition is that the mental and physical events occur in distinct moments which are simultaneous with one another. If this supposition, subsequently to be dismissed as unwarranted, be granted, it follows, not that the moments of time are without connexity, but that mental events and physical events intensify a single moment in different ways. If two moments are not connected by the relation of before and after, and if they are not in the relation of part to whole, they are not distinct moments of abstract time.

The relation between the moments of time is asymmetrical ; i.e., if a moment is before another, that other is not also before it. This means that no circular arrangement of moments can constitute a time series except so far as the circle is not completed, or if, upon the completion of the circle, the beginning as returned to, was differentiated from the beginning, as the original starting point. In the former case, the series of moments would be in a linear arrangement as required, but time itself would cease with the completion of the circle ; in the second case, the moments of time would be arranged linearly also, since the differentiation of a given moment as the beginning and as the end would be a differentiation which would constitute it as an entirely different moment and make it, together with all the moments that follow it, successive to, and not identical with the moments that went before. Theories of eternal recurrence do not deny the asymmetry of the relation of before and after, connecting the moments of time. They suppose that the history of the universe can be duplicated, detail for detail, in the very order in which it pre-

viously occurred, a supposition which is consonant only with the unwarranted denial of the fact that the future is always novel. A "circular theory of time" is a theory of eternal recurrence which characterizes the moments in terms of the nature of the events occurring at those moments. But such a theory cannot maintain itself. There is a moment now which is present and other moments which are future. Those future moments differ from the present moment in that they have occurred once less than it has. The present moment in becoming present must have achieved the trait of having "occurred the  $n$ th time" if it is to be differentiated from itself as not yet having occurred the  $n$ th time, i.e., as being one of the future moments of this epoch. But as so differentiated it is a new moment in a linear order, asymmetrically related to all previous and succeeding moments.

The relation between the moments of time is transitive: i.e., if any moment precedes a second which precedes a third, the first precedes the third as well. According to some physicists, however, there are cases where an event is before another which is before a third, but the first is not before that third. But it is because they too recognize that there is no meaning to a time series unless the moments are connected by a transitive relation, that they hold the doctrine that there are many different time series, within which the moments are connected by a transitive relation. To deny that there is a transitive relation of before and after holding between moments is not to deny that such a relation is essential to time, but to insist on it, and to draw the consequence—to be examined below—that there must be many different time series.

It seems then that both those who acknowledge but one time series and those who embrace many, are agreed that one and only one member of a pair of moments is before (or after) the other, and that if that other is before (or after) a third, the first is before (or after) that third. Since all entities related by a connexive, transitive, asymmetrical relation form a series, time must consist of a series of moments.

But to say this is to specify only some, but not all, the essential characters of time, since it does not permit of the differentiation of time from other kinds of series. There are logical, valuational and numerical series; the presupposed is logically, not temporally prior to the conditioned; the better is prior in value, not in time, to the worse; the natural numbers, in their ordinary arrangement, are quantitatively, not temporally, before and after one another. So far as the moments of time are viewed solely as being before and after one another, the relation that connects them could just as well be designated as the relation of logical priority, virtuous superiority or numerical inferiority. The time series, however, differs from other series in at least five respects; the entire series is extended, the moments are homogeneous, contain a distance, form three exclusive classes and pass.

The common image of time is a line with a direction, since such a line shares with time the character of being an extended stretch having subdivisions, internally undifferentiable from one another, in the order of before and after. Space and time differ from everything else in the universe by their extendedness and the homogeneity of their subsidiary parts. Their extendedness consists in the fact that their subsidiary divisions are such that each one is the relation between two others as well as a term to be related by some other. Any domain whose members consist of elements which are both the terms related and the relations relating is a region of extension. Though there are many conceivable varieties of extended fields, there are ultimately only two basic kinds—those in which the relations are asymmetrical, as in time, and those in which they are symmetrical, as in space. Each moment of time is related to every other by an intermediary moment which, because it is also a moment related by those others, holds them apart and unites them as well.

An extension is homogeneous so far as the properties of any one term are identical with the properties of any other. All empty spatial and temporal stretches are imaginatively

divisible into parts all of whose properties are the same, so that the relation which unites has the same properties as that which is united. The relation between two moments of time is another moment; and conversely, a moment between any two moments relates them. Time does not consist of moments and relations, but only of moments which relate and are related by moments, in the order of before and after.

A distance is a measured extension. Each kind of extension has a unique unit of distance, the capacity to be filled with concrete objects which possess the same formal properties as the abstract extensions. Time is measured by clocks and space by rulers, or what is the same thing, clocks are spatial objects with marked asymmetrical transitive characters and rulers are temporal objects with marked symmetrical intransitive characters. All measures can be brought into one-to-one correspondence with the distances of space and time — and here Kant was on the right track in his analysis of arithmetic and geometry — because the stretches of empty time and space are the least intensive, the most abstract of all possible units, and all others can be viewed as specifications of them. It is the realization of the program for the reduction of all distances to those of either space or time that must constitute the goal of any rationalistic philosophy. Classical rationalism came to a sorry end, not only because it was internally incoherent but because, for it, extension and distance were irrationals, incapable of definition, whose presence was the mark of mental confusion or of the arbitrariness of a supposed original designer.

The series of the moments of time differs from logical, valuational, numerical and similar series in that it is extended and homogeneous, and thus contains a unit of distance applicable to both terms and relations alike. But these characters, though they suffice to differentiate time from these other series, do not suffice to differentiate it from the image of time in the shape of a line with a direction. There must evidently be other features not yet discussed, which are essential to

time. One of these is that the moments of time form three distinct and exclusive classes — or better, form three stretches, a past, a present and a future. To say that time is made up of such stretches, is, however, to say something that could be said about a vectoral line which had three stretches marked off by different colors. Since such a line would have every trait so far demanded of time, and since such a line would nevertheless have its three divisions co-ordinate and fixed, it is evident that the nature of time has not yet been revealed. To specify the moments of time as being past, present and future is not to characterize real time, but merely possible time, a time still imbedded in eternity. To differentiate time from its representation in the form of a line it is necessary to acknowledge the fact of passage. Unless each moment in turn possesses and sheds the character of being future, eventually assumes and loses the character of being present, and finally adopts the character of being past, there is no time.

But the moments of time must pass with respect to something, and something must pass with respect to the moments of time. These expressions are interchangeable because passage involves a contrast between two factors, either one of which can be taken as fixed with respect to the other. But just as there can be no change in time and no flux of actualities unless there is something persistent in the other, so there can be no passage in either unless there is also a passage in the other. If there is a change in the world, there is a passage of moments, and if there is a passage of moments there is a change in the world. An event can pass only if it is correlated with a moment which is but will not be, just as a moment of time can pass only if it is correlated with a state which is but will not be. A passing fact for which there was no passing moment would be in a world in which there was change but no time. A passing moment, on the other hand, for which there was no passing state would be a moment which passed without having anything to pass.

An accurate view of time must, if it is to include all these

observations, show how it is possible for the things of the world to persist while the moments of time pass, time to remain while things change, and both the moments of time and the changes in things to pass together. The coalescence of all these into a single coherent account, consonant with other empirical and metaphysical observations, constitutes the problem of time and change.<sup>2</sup>

## 2. *The Reality of Time*

THE "receptacle" or "absolute" theory of time, as developed by Plato, Plotinus and Newton holds that time is a constant moving image of eternity, indifferent to and independent of the motions or changes that do in fact occur. But there is no passage of time, and thus no time at all, unless there is something outside of the moments of time in terms of which the moments pass. Suppose that the series 1, 2, 3 represents the moments of time and that 1 is present while 2 and 3 are future. There will be no time unless 2 becomes present while 1 becomes past and 3 becomes present while 2 becomes past. If "presentness" were an intrinsic character of the moments, all three of the moments would, contrary to the hypothesis, be present together. If, therefore, "presentness" is a character of a moment independent of anything outside the time series, it must be a character associated with the position which a moment occupies, and which is occupied in turn by the others. The moment 1, then, must abandon that position, if there is to be time, and the moment 2 must occupy it when 1 abandons it. But if the order of before and after is not to be disrupted, 1 must still remain before 2 even when 2 occupies the first position, originally possessed by 1. There must then be a position "before the first" that 1 occupies when 2 is present. But the series in which 1 is in the first position is a different series from that in which 2 is in the first position. Let us term these two series the series A

<sup>2</sup> Within the compass of this paper it is impossible, of course, to deal with the entire problem. I shall deal only with some features of it.

and the series B. Series A is itself before the series B and in the series A-B must occupy the first position. For 2 to be present, series B must occupy the position of series A and series A must occupy the "before the first position." But the series in which A is in the first position and B is in the second is a different series from that in which A is before the first and B is first. If B is to be in the first position, the series where A is first and B second must be replaced by one in which A is before the first and B is first. But the two series, A first, B second; A before the first, B first, constitute a new series in which one of them is in the first position and into which the other must pass, and so on without end. If we refuse to acknowledge something outside the series of the moments of time, we must acknowledge it outside the series of that series, and if not there, then outside the series of the series of series, and so on. The moments of time must successively occupy a position determined by something outside of the time series. And that position, if it is to be occupied by each of the members must persist so that they can replace one another in it. That external determinant in terms of which the moments of time are successively present moments I term the Now.

What is true of the moments of time must be true of the sequences in a changing world. Successive facts are before and after and must replace one another in a persistent locus which is outside them. That position which a succession of happenings occupies I term the Present. Just as time cannot pass unless there is an enduring Now, so things cannot pass unless there is an enduring Present.

To acknowledge the Now is to deny Absolute Time. Absolute Time, however, is to be distinguished from Universal Time, though those who have accepted or rejected the one have usually accepted or rejected the other. I accept Universal, but reject Absolute Time. Absolute Time is the time that flows regardless; it could be even if there were nothing else. Universal Time, on the other hand, is the time which

encompasses all real objects. The opposite of Absolute Time is Concrete Time, the opposite of Universal Time is Singular Time, though contemporary literature calls both opposites "relative." Even if modern physicists had not been forced to recast Newton's physics, his doctrine of Absolute Time would have had to go as an irrelevant concept for a scientist and as an unintelligible concept for a thinking man. It was not abandoned, however, until and because Newton's doctrine of Universal Time had been rejected in favor of multiple singular times, for somewhat recondite and not altogether cogent reasons. But it is possible to hold that there are multiple singular times each of which is absolute (were absolute time at all a tenable notion), and it is possible to hold that there is a universal time dependent for its being on the presence of something in the world. The choice between absolute and concrete times is that between time as transcendent and as immanent; the choice between universal and singular times is that between a changing universe and changing independent universes. But time is immanent in the universe. It can be neither absolute nor a multiplicity of singulars. Concrete times are multiple and absolutely unique, but they are not singular, since each is but an intensification of a common universal time.

The doctrine that there are multiple singular times is one of the consequences that, in recent years, has been drawn from the "shadow" theory, though its original protagonist, Aristotle, explicitly affirms the universality of time by fastening on the eternal, regular circular motion of the heavens as the locus of time, *par excellence*. To acknowledge the Present, however, is to deny the shadow theory whether in the form defended by Aristotle or in the form defended by Einstein. That theory takes time to be an abstraction from concrete passage, possessed of no reality of its own. It is a view which cannot acknowledge a real time apart from changing actualities and thus must deny the reality of a future or a past, with a consequent incapacity to give sig-

nificance to the present ; it is powerless to explain the fact that abstract time passes with a constancy which is indifferent to the rapidity of phenomenal changes, and that despite the diversity of the rates of change of entities in a coexistent world all of them move into the past together, whether or not there be a knower capable of abstracting a common measure from them all.

The absolute theory of time ignored the truth that time is rooted in reality. It must find a Now in terms of which time passes, and eventually has to have recourse to an eternal Deity for which time is the moving image. The shadow theory of time, on the other hand, ignored the truth that time has a nature of its own. It must find a Present in terms of which things pass and eventually must have recourse to some constant in terms of which all the supposed diverse singular times can be measured. Kant's refutation of the problematic idealism which is doubtful about the objective reality of time has as an inevitable consequence the rejection of the shadow theory as well. Time and actual change are inseparable, each with a nature of its own, co-ordinate because they are parts of a reality which includes them both.

It is the taking on and off of the characters of future, present and past that is the source of most of the denials of the reality of time, whether it be viewed as concrete or empty, as absolute or singular, as an independent reality or as an abstraction. In fact, since it is the passage of its moments that differentiates time from all other series, it must be with regard to this feature that theories of the unreality of time, in contrast with those regarding space, must find their justification.

Out of the welter of denials of the reality of time a hierarchy of five basic objections can be distinguished. According to the first, "a moment of time, if it has any being at all, must have a nature describable without recourse to anything outside of time. Yet the moments of time achieve the characters of being past, present and future only with reference to something outside of time, so that past, present

and future cannot be essential features of time. Since there is no time without past, present and future, these characters are at once essential and inessential characters, so that the very concept of time is lost in contradiction." But an essential character is not necessarily a monadic character. Part of the being and nature of every actual entity consists in its relation to something else in virtue of which it has features indispensable to it. Dependence on the environment makes a creature not less but more of an animal, and its very essence involves the possession of traits, such as teeth, which are what they are only because the creature is in relation to what is outside it. In the same way, the moments of time have as part of their nature characters which are theirs only because of their relation to the Now which lies beyond them.

The second objection pursues this difficulty further. It says in substance, "It may be that the character of being present is part of the relative character of this moment. But how can it be of the essence of that moment when it is immediately replaced by the character of being past, which is an opposing character?" According to this objection, no trait can be essential unless it is a fixed and irrevocable character. It supposes that traits that vary are idle adjectives adhering precariously to a permanent and essential substance, a view which necessitates the supposition that changing objects either are not substantial or that they are totalities of discrepant traits. All traits are either public or private; those which are public are relative, some of them being permanent, others transient. The permanency or transience of the public trait has nothing to do with the permanency or reality of the entity to which it belongs. The permanent nature of a specific feline does not consist in the possession of the character of being a domesticated animal, but in its inward capacity to do what cats do in the situations that cats get into. Similarly, a moment of time has a nature which permits it to possess relative impermanent characters which express its inward nature. A moment is a passing

extension and to it necessarily accrues the relative characters of being future, present and past.

The third objection drives the second still further. It says, "According to the reply to the two previous objections, past, present and future are part of the nature of the moments of time. They are mutually exclusive traits, yet each moment possesses them all — a manifest contradiction. It will not do to say that a moment is future, present and past in turn. Let, e.g., 1, 2, 3, represent a series of moments of time in which 1 is past, 2 is present and 3 is future. Were that series future, the moments 1 and 2 would be future; were that series past, all the moments would be past; it must therefore be a series which is present. In that present series 3 is a future moment. As such it will be present and past in a future time. 3 is therefore now future in a present time and present and past in a future time. But there is only one time and in it moment 3 is at once past, present and future." The objection falsely assumes that the whole of time is either past, present or future. But it is none of these; it has no temporal characters. There is no present Time in which 2 is a present moment and 3 is future while 1 is past; there is only Time and in it 3 is *now* a future moment, 2 is present and 1 is past. 3 is not a present moment in a future Time; it is a future moment now when 1 or 2 is present. We do say that the *time* when 3 will be present is now future but that is but a way of saying that the moment 3 is now future and will be present. A moment is future only with reference to a moment as present. 3 has the character of being future now; it will be present later, in the same Time; it is not now present in a now existing future Time.

"But this answer," continues the fourth objection, "only drives the issue back a step. How is it possible for moment 3 which is future ever to become present? Such a change takes place in time. But the time in which 3 is a moment is a fixed series of moments in the relation of before and after. The change in the character of the moments must then take

place in another time, and so on without end." This objection supposes that to pass is to pass with reference to Time. But time is not the locus of a multitude of ordered "present" moments, each of which is to manifest its presentness through a change in the whole of time. It is a series of moments in the relation of before and after, each of which is capable of being determined, by virtue of a change in its reference to the Now, as past, present or future. The passage of moments is not in terms of time, but in terms of something other than time.

"But nothing," the final objection continues, "can become present or past. The future and the past altogether are not, so that if the future could become present and the present past, a being would issue out of non-being and vanish into non-being again. But nothing can come out of and go into nothing. There can only be an eternal present. Yet that present, if it have any temporal extension at all will itself break up into a past, present and a future. There is no time but the present and that is only a point which has neither duration, motion, or extension and thus is not time at all." But time does not consist of a set of moments, lined up like soldiers waiting to file into a yet undefined world called the Now, and in which they have an instantaneous life and death. Time consists of three undivided stretches, bounded with respect to one another — an indefinite past, a momentary present and an indefinite future. The future is potential, actualized by something concrete, becoming as a consequence a discrete, bounded actual present moment, part of a real being which, in contrast with it, is the Now. Future time is indeterminate and real, but not actual; it is undifferentiated into moments, continuous and without content. The moment of present time is determinate, real and actual; a finite extension ingredient in whatever individuals there may now be. The whole of past time is determinate and real, but not actual. It is the intensive depth of actualities as preserved in the form of an organic memory within whatever

individual there may be. The process of time is thus not a shift from non-being to being and back again, but the actualization of a potentiality, the becoming distinct and determinate with a subsequent loss of distinctness. The accumulation of the past and the shift of the mode of actualization in the present makes the present forever new, while the future remains unpredictable because indeterminate and the past forever hidden, because not actual.

### 3. *Time and Passage*

THE moderns pride themselves on the fact that they take time seriously. It is quite apparent, however, that they have taken it too seriously, or what is perhaps the same thing, have not taken it seriously enough, for it has run away with them. The clarion cry is that time is integral to the universe. That thesis, which is as old as Aristotle, is today, however, read with a new accent so as to mean with Heraclitus that there are no things which persist, but only events which perish as they occur. Only passage for the moderns is real, a passage which somehow becomes decorated at each moment with idle characters, deceiving the unwary into believing them to be characters of real persistent things. For them nothing is persistent or recurrent — except perhaps the truths which their theories alone embody. This wisdom, some of them think, is the product of modern science; others with more justice find it in Plato or in his Kantian disciple, Schopenhauer. Its origin, I think, is to be found at the dawn of modern philosophy; but whatever its source, it can be embraced only by losing most of what we know to be true.

Descartes, the putative father of modern philosophy — the mother, as is too often forgotten, was scholasticism — stated the problem of persistence clearly and briefly. "All the course of my life," he observes, "may be divided into an infinite number of parts, none of which is in any way dependent on the other; and thus from the fact that I was in existence a short time ago it does not follow that I must be

in existence now, unless some cause at this instant, so to speak, produces me anew, that is to say, conserves me. It is as a matter of fact so perfectly clear and evident to all those who consider with attention the nature of time, that in order to be conserved in each moment in which it endures, a substance has need of the same power and action as would be necessary to produce and create it anew, supposing it did not yet exist; so that the light of nature shows us clearly that the distinction between creation and conservation is solely a distinction of reason." [3d Meditation, Haldane and Ross translation.] For Descartes each thing perishes with the passing moment. All supposed persistent things are, for him, only successions of distinct and independent beings, each of which is a duplicate of the preceding one, with only that modicum of difference which expresses the fact of change or motion. Though an entity at one moment is distinct from itself at another, thanks to the conservative actions of God, it is recreated, he thinks, sometimes exactly as it was before. Though Descartes denies that anything really persists, he is nevertheless able to affirm that things do in fact recur. But God is only a hypothesis, defining the locus of a metaphysician's distress. If all beings are momentary, recurring only because of God's arbitrary act, what makes God persistent so that He can recreate? Descartes answers that He conserves Himself. But God does not have to conserve Himself if he is in fact eternal; and if He is in time He must perish with the passing moment, since each moment is, for Descartes, external to the others. God, for Descartes, is an enduring being, depending on no power beyond Himself for His persistence, because He has, in fact, been placed beyond the possible ravages of time—but this is but a way of saying that that which persists is really eternal.

Deny, with Hume, the existence of the Cartesian God, and the world breaks up into a series of atomic entities, completely cut off from one another. We shall then have lost all connection between the present and the future and shall

find ourselves impotent to account for the occurrence of the next moment and of any entity in the future, whether it be the same or different from what had been before. A Cartesian separation of an existent into momentary beings, without a Cartesian God to guarantee the continuance of the sequence and the possession of an identical core in the successive members yields a world in which anything might happen (including the occurrence of a new world in which entities actually persist); a world which now persists by virtue of a miracle performed by no one. This is the last word of a rigorous empiricism; unfortunately it is compelled to deny every empirical fact.

Take the Cartesian God, on the other hand, with Leibniz, and multiply him times innumerable so as to achieve a universe of isolate, unrelated beings. We shall then have a set of entities which not only persist, but unfortunately last forever. We have now moved to the opposite extreme from Hume and cannot find anything real that perishes. If we people our world with Cartesian Gods it becomes a place where everything real is eternal and nothing is related. But we shall then have to deny that relations are real and subject to change, that there is any causal action exerted by one entity on another and that there are any changes in public characters and spatial positions.

Take out the time from within these multiple Cartesian Gods and relate these Gods to one another and, with Newton, we shall obtain a universe of eternal atoms which never change, but which can be caught in changing relational situations. No real thing, on this view, really persists unless it is eternal; but the relations between them may persist forever or change constantly, they alone being in time. (Newton, after Descartes, saw that if the relations of atoms to space change in a constant way, the relation of these relations will itself persist—a proposition which is incorporated in the first law of motion.) This is a view which can acknowledge the fact that there are both persistents and mo-

mentary beings in time, but unfortunately it is compelled to take such beings to be the relations between or the aggregates of eternal entities, never true substantial individuals.

Individuals change with reference to one another, not because they mirror one another, or because an instantaneous physical force drives them into harmony, or because they remain in the same state while a space-time matrix undergoes an internal disturbance, but because they and their relations are caught in a single ubiquitous fact of passage. It is by overemphasizing this passage that we reach the standpoint of the peculiarly modern theory of time.

If with Schopenhauer and Bergson we place a Cartesian God underneath the universe, depriving it of its wisdom, so that it becomes an indivisible, blind surging movement which sees not where it goes and which no reflecting being can know, the moments of time will no longer have to be viewed as external to one another, as Descartes supposed. But for this vitally important gain, we shall have to pay the price of supposing that every discrete entity is necessarily illusory. An indivisible surging movement which is the basic reality makes everything else an illusion — an illusion which, because excluded from time, must be eternal in a transcendent mind or in some other kind of Platonic heaven. We metamorphosize our Cartesian God into the basic fact of an indivisible creative urge only by giving up the reality of every transient fact that we might possibly know.

Bestow pulsations on this creative urge and with Whitehead we can say that at each moment it convulsively produces discrete real entities within itself — entities, however, which last but for a moment. Whitehead has made more sense of the theory that passage is fundamental than any of his contemporaries, for he alone can admit that things and relations are real and that they change with reference to one another. Yet he has made no advance on Descartes' discrete occurrences. In fact, he has lost even the recurrent which Descartes was able to acknowledge. For him each entity is

entirely other than the entity that occurred the moment before. He thinks that a number of them sometimes, by a happy accident, form a historical society having components which are somewhat similar to one another and which will give the appearance of a real persistent, but he never admits that a single entity ever lasts more than a moment. If the fundamental fact is passage, things can be but momentary accidents decorating events, and no matter how old a thing may be, we are bound to say it is entirely new. We worship the rhythmic fact of passage only by giving up the persistence of whatever there may be.

Descartes, Leibniz, Locke, Berkeley, Hume, Kant, Schopenhauer, Hegel, Einstein, Bergson, Alexander, Dewey, Bradley, McTaggart, Whitehead and their disciples — in fact, almost every philosopher and cosmological scientist in modern times — share a common assumption which is erroneous. They assume that the real objects in the universe are completed individuals at every moment of time. For all of them it is an axiom that to be a discrete entity in time is to be something which perishes with the passing moment, because inescapably confined within the span of that moment. The differences between these thinkers result largely from their diverse decisions as to whether or not every entity is in time and thus as to whether or not there is any element in the universe which actually persists. All of them, consciously or unconsciously, seem to have embraced the principle: any individual in time can live but for a moment.

But temporal, enduring objects confront us on every side. The fundamental fact cannot be passage, but plural things which change while they persist. To deny this truth is to commit the *fallacy of essential completeness* and suppose that entities either have non-temporal boundaries and are thus eternal, or have temporal boundaries and thus must perish with the passing moment. It is remarkable that Whitehead, who has pressed home the more difficult and less plausible argument that it is a fallacy of "simple location" to suppose

that there are entities which have a definite place in space or time apart from a real relation to other regions, should have been so blinded by his doctrine of the space-time receptacle as to be unable to recognize the more fundamental and obvious fallacy of essential completeness, of which his own was a specialized and dubious instance. Though he expressed the fallacy of simple location as if it had application to temporal characters, he never saw that he should then have maintained that no things can be viewed as merely present. Instead he takes each thing to be a momentary being, containing *within itself* a reference to an external past and an expectation regarding an external future. The fallacy of simple location then comes to mean not that it is false to suppose that entities are in a single region of space or time but that it is false that they are in such regions without an internal reference to others. He is compelled then to say that a thing is located in a single moment of time, though it points beyond that moment. But pointing does not make it persist and Whitehead is thus here still one with Descartes and Hume, whom he so valiantly tried to correct.

*To be is to be incomplete*; an actuality with its equilibrium outside itself; a being whose essential boundaries lie somewhere in the future. The fundamental fact is not passage, but individuals which can change while they persist and can persist because part of their being lies outside the confines of the present moment. They perish only by being divided or by being assimilated by others; persisting otherwise because at no one moment have they become real enough to enable them to vanish completely.

# CAUSALITY

BY S. KERBY-MILLER

THERE are two fundamentally different views of causality, involving different typical instances, different ideals of causal explanation, different accounts of the process of discovery and confirmation of the causal relation, and involving, finally, different theories concerning the function of concepts and language in our knowledge of causation. The one, which we shall call the 'regularity theory' finds its best instances in the laws of physics, the other, which we shall call the 'intrinsic connection' theory finds its best instances in judgments based upon an introspective analysis of changes in consciousness. According to the regularity theory a universal correlation between two identifiable events constitutes complete causal explanation; according to the intrinsic connection theory, a mere correlation, no matter how universal, is hardly more than a paradox, at best it sets a problem. On this view of causality the perceived and felt connection between two states of consciousness may supply the very ideal of intelligibility and understanding, on the regularity theory such a supposed insight is the merest superstition, and the supposed connection something which is to be established or not by repeated experiments and made intelligible by being expressed in terms of a formal law applying to the behavior of animal bodies. Those who hold this theory would claim that we know most of the physical world and least of man, those who hold the other would claim precisely the reverse. One claims that the maximum knowledge of causality is obtained in direct insight into immediately experienced happenings, in the full and discriminating experience of individual occasions; the other claims the maximum knowledge to lie

in knowledge of the application of abstract conceptual systems. Abstract systems and conceptual structures are for the intrinsic connection theory a mere means of aiding insight, of sharpening discrimination, of analyzing the experience of particular occasions, for the other particular occasions afford merely marks and signs for application of abstract systems. For one, successful prediction constitutes the criterion, even the essence of knowledge, for the other it is without much theoretical importance and attaches primarily to low grade knowledge. The ideal of one is to achieve the greatest degree of calculability possible, with symbolic instruments of the least complexity, the other to give discernment and insight. Language on the regularity theory is a symbolic mechanism which is useful in proportion as its rules of syntax are autonomous and simple and can be manipulated in such a way as to make predictions of the spatial and temporal order of sense data. Mathematics, consequently, appears to be the ideal language. Language on the intrinsic connection theory is primarily a means of expression. Its function is to set before one the realities concerned, or to reproduce experiences with insight, much as an artist does in a painting. In proportion as its syntax is autonomous, in that proportion the language is inexpressive. Its function is to recall, not to be a substitute for, the realities. As language it points to the realities; as thought and judgment it reproduces their essential character with discernment. The intelligence is satisfied where there is insight into intrinsic connections in the realities before the mind. It is in these connections directly discerned that causality is best known.

The differences between these views of causality are so fundamental that the proponents of each seem to be engaged on different enterprises with different kinds of things, and this is, of course, in part true. Both, however, claim universal application for their views, and each claims his facts to be typical and his criterion ultimate. I cannot attempt, within the limits of this essay, a full analysis of the issues

involved in this conflict of these two views of causality. It seems to me that each view has brought to light fundamental facts and considerations which cannot be accounted for by the other, and that the solution of their conflict cannot lie in the simple reduction of one to the other. In this essay I shall attempt to show that there is a class of judgments asserting causal connection which cannot be properly interpreted on the regularity view and to indicate how such judgments are presupposed by the regularity view itself. This is, of course, a very small part of the general problem and represents a one-sided approach, but there has been in recent years such an emphasis upon the interpretation of the physical sciences and so strong a tendency to regard these as typical of all knowledge and the proponents of the regularity view have been so active and successful in that field, that it behooves anyone who is attempting to show how the facts and considerations found in the intrinsic connection view of causality supplement those of the regularity theory, to show that there *are* such facts and considerations. This, then, is the primary purpose of this essay.

The phrase 'intrinsic causal connection' may not be fortunate, and I must ask the reader's indulgence with it until I can define its meaning more exactly. It is intended first of all to stand in sharp opposition to the theory that the causal connection is known merely as 'regularity of sequence,' and that verification of a causal relation must lie solely in an inductive confirmation of some law of correlation. It implies that there is a knowledge of causality which is not reached by an inductive process, which is not merely knowledge of a regular sequence and which we do not seek to confirm by an inductive process. Thus it is maintained that the causal relation asserted in such a proposition as 'I believe this mathematical proposition *because* I have just seen its demonstration,' does not derive its meaning or its confirmation from inductive evidence. I do not base the judgment that my present belief in this mathematical proposition was caused

by my seeing its demonstration, on a law of correlation between believing propositions and seeing them demonstrated. The fact that such a judgment does not derive its meaning from some law or regular sequence is evident on the face of it, but we may note further that there are cases in which this judgment may carry a much higher degree of assurance than the general law—'seeing a demonstration is always followed by belief in the proposition demonstrated.' I may be certain in the particular case while believing the general law does not hold. It follows that our assurance is not based on the law, and it follows likewise that the meaning of the statement cannot be derived from the law.

There seems, then, to be a meaning in 'cause of' in such judgments which is not simply a derivative of uniform sequence, and that there is such a meaning is further suggested by the way in which such judgments are confirmed. If I ask myself, Was 'seeing the demonstration' the cause of my belief? I proceed at once to recall as clearly as possible the circumstances, and when I do succeed in doing this I decide the case by a discriminating and analytic inspection. In the most favorable cases, moreover, I am very disinclined to admit the possibility of substantial error. It is only when I fail to recall the circumstances clearly that I resort to a general rule and decide, e.g., 'It is the sort of proposition I would not have believed unless I had seen the demonstration.' This judgment in effect asserts only that there was *probably* a causal connection.

A direct method of deciding the presence of causal connection is used in many cases involving motivation, purpose, decisions or assertions based on evidence of various kinds, etc. In such cases our method is to scrutinize with care our memories of the particular occasions involved. Our judgments are based upon a direct inspection and claim direct insight into the causal connection present in a *particular* case. It is the basis of these judgments which I have in mind when I speak of 'insight into causal connection.' Its existence

seems to be implied by (1) the existence of some particular judgments of causal connection which have a degree of certainty greater than the general rule, (2) and the fact that these judgments are based upon direct perception and that the *direction* of confirmation is toward inspection of the individual occasion without reference to inductive generalizations.

There is at least so much positive evidence for believing in a direct acquaintance with causal connections. Before considering this and other evidence more in detail I wish to consider briefly three *a priori* objections: first — that to claim direct insight is to claim infallible knowledge; second — that the fact that causality is an interpretative principle or category is inconsistent with direct perception of the causal relation; and third — that we cannot perceive causal connections within the limits of the 'specious present.'

It is sometimes argued that to claim that a judgment is based upon direct insight involves the claim that it is infallible, and that since there are no judgments about existence which are absolutely infallible, there can be no direct insight. In spite of this consideration there do in fact seem to be some judgments which are based on direct insight and are yet subject to error. Such judgments as 'This is Chinese red,' 'This is brighter than that,' 'This is to the right of that,' 'e<sub>1</sub> is temporally before e<sub>2</sub>,' where the terms are objects immediately present, are based upon direct insight and are yet subject to error. They contain two factors both of which admit of uncertain determination. The general terms may be inexact and ambiguous in various degrees and the presented complex may be such that it is difficult to determine by direct inspection whether or not the universals or relations are exemplified or not. Thus I may not recall clearly just what is usually meant by 'Chinese red' or even exactly what I generally mean by it, and though I see very clearly just what this colour before me is, my judgment concerning it may be uncertain on that account. Again my perception of the presented colour may be wavering and uncertain. Similarly

I may have a rather uncertain idea of what 'to the right of' means (this is common in children) and though the spatial position of the objects before me may be clear, I may be mistaken in calling it 'to the right of.' On the other hand, with a clear notion of what is meant by 'to the right of' I may be uncertain whether this (A) is to the right of that (B) or not. A and B may be dots rather far removed from one another and very little off a perpendicular. Similar conditions hold for the other examples. They all admit error from two sources—lack of precision in the concepts or language and lack of precision in the perception. None the less, where the subject of the judgment is 'the content of experience' verification must take the direction of direct scrutiny. No mere analysis of the concepts involved and their implications will in itself determine the truth or falsity of these judgments. The uncertainty of judgments based on immediate inspection may, however, be exaggerated. In favorable cases they may carry an assurance which is beyond reasonable doubt. In making the judgment 'This is red' I may have a very definite idea of what I usually mean by red and the colour of this object may be vivid, and I may intend only a rough correspondence. In such a case, the judgment approaches certainty, and such judgments are not unusual. I think we can generalize and say that judgments based upon direct inspection may approach certainty in cases where there is no substantial ambiguity of terms and where the circumstances judged about are *favorable* for determining the questions.

The second *a priori* objection might be stated thus: causality is a category by means of which we interpret our experience and give it order. Thus when we predicate the causal relation we are asserting that the data conform to the conditions laid down *a priori* by the concept, and this is an act of interpretation not of direct knowledge or insight. On this account direct knowledge of the causal relation would be impossible—by definition. I can here only set down briefly

and somewhat dogmatically the considerations which I believe meet this objection. The category of causality defines, primarily, what we mean by a causal *law*. It is *a priori* in the sense that we assume that all nature is ordered by law. The category then will apply strictly only to terms which are universals and will be in the form: 'Events which have universal A cause events which have universal B.' Though it is not inconsistent with the category to suppose that there may be some intrinsic or logical relation between A and B, the minimum essential categorial condition of A's being regarded as the cause of B is regularity of sequence. Thus the minimum ground I can have for asserting a causal relation between two events  $e_1$  and  $e_2$  is noting the presence of some universal A in  $e_1$  and the presence of some other universal B in  $e_2$  and the knowing that A is always followed by B. Contrariwise, whenever I assert that  $e_1$  is the cause of  $e_2$  I mean that they are related thus by a causal law.

But here, of course, we come to a question of fact; i.e., is this the proper analysis of what I mean in every case in which I assert the causal relation between two events? Such a judgment as 'I asserted the proposition because I had just seen the demonstration' does not, I am maintaining, yield to such an analysis and this, I have argued, is evident from the fact that we may feel certain about the particular instant and doubt the general law.

The first thing to note is that there need be no conflict between them. I can in every case believe, on apriori grounds, that whenever there is a causal relation there is a causal *law*, and attribute my failure to find the law, in any case when I am otherwise assured of a causal relation, to an inadequate analysis of the universals in the circumstances, together with an inadequate knowledge of the possible regular sequences between these universals. This, of course, excludes the possibility that by 'causal relation' I mean merely 'an instance of causal law.' If this is what the causal relation means by definition then every judgment asserting a causal

connection between individual events without reference to a law is nonsense, and this, as I shall show, condemns a great many judgments ordinarily regarded as important. It seems to be simply a fact that we may be fully convinced of the presence of a causal relation between two events without actually specifying any characters or combinations of characters within the true events which are related by a universal causal law.

Assuming that there are such judgments, it is clear that the idea of causal relation, as it occurs in them, cannot be defined in terms of the category only, but must derive at least some of its meaning from designating a directly inspectable relation. And we must indicate how this directly inspectable relation is related to the category of causality. I believe we can see what this relation might be by considering another instance of the relation of a directly inspectable relation to categorial conditions. There is in our notion of space two distinct elements—the geometric and the perceptual, the one universal and conceptual, the other particular and factual. Some judgments concerning spatial relations involve inferences based upon the postulates of geometry, others express relations which are directly perceived. Such a judgment as 'A is to the right of B' may be purely perceptual or it may be the result of an inference based upon a geometric postulate, e.g., A is to the right of C and C is to the right of B, thence (since spatial relations are transitive) A is to the right of B. The perceptual judgment is, however, the basic one. The postulates of geometry are in nowise prior to it but on the contrary are geometric (as opposed to merely logical) because they involve a tacit designation of perceptual spatial relations. Our word 'spatial' primarily designates apparent perceptual relations and the a priori geometric conditions must conform to them if they concern spatial structure. Geometry does not lay down conditions to which spatial perception must conform but vice versa. We can conclude that geometric propositions always hold of our spatial perception

only by induction or by some species of generalizing insight. Geometry does, however, lay down those general conditions for *inferring* non-present spatial relations. Generalizing from the relation of direct perception of spatial relations to geometry, we may argue that it is possible to have a logical system which applies to directly perceivable relations. This system would be the basis for all inferences concerning the existence of non-perceived relations and it would lay down the conditions of the validity of any such inferences. The concepts in such a system have, however, a dual nature; they are partly logical and partly designative. (They belong to a logical system having a specific interpretation.) The system, correspondingly, has a dual verification, as descriptive and as predictive. Some but not all of the relations in the geometric causal or temporal systems must be directly verified, and be seen to be descriptive of the perceived relations, e.g., the relation of transitivity is characteristic of perceived spatial relations. And the system as a whole must enable us to predict future spatial relations. I may infer that the world is spatial because I can make correct predictions on the basis of a geometry which conforms to the relations I directly perceive. In asserting that the world is spatial, I mean that it is characterized by the same kind of relations which I directly perceive. Similarly, it may be that when I infer causal relations on the basis of inductions in accordance with conditions implied by the category, I am attributing to nature the sort of relation which I directly perceive to exist between particular events. The category specifies conditions under which I am justified in inferring non-present causal relations. These considerations are, I think, sufficient to meet the objection that the existence and function of the category of causality is *incompatible* with the direct perception of the causal relation and also to indicate how positively they might be related. Namely, the category lays down the conditions for inferring causal laws, and, on the basis of such laws, the presence of causality in

particular instances where the causal relation is not directly perceived; but the concept itself is partially designative in meaning and indicates the sort of relation which is directly perceived. I have not here, however, attempted to do more than indicate this possibility for the purpose of meeting the *a priori* objection. I shall return to consider the problems involved, but I shall not be able even then to give a full analysis of the relation of the category to the perceived relation. I do not hold that the category is strictly a generalization of the perceived relation. There are elements in the category not present in the perceived relation, e.g., the law, same cause same effect, could not be discovered from an analysis of any particular causal connection. Nor could, on the other hand, all the laws of geometry be regarded as generalizations made from the analysis of what was contained in a perception of a particular spatial relation.

The third *a priori* objection I shall consider concerns the possibility of perceiving a causal connection within 'the specious present.' The objection may be stated somewhat as follows: Any obvious and microscopic example of causal connection spreads over more time than can be included in the specious present and so depends upon memory. Thus the inspection of the causal relation would be an inspection of the relations between memory images or at best between a memory image and a present event. But since the actual causal connection does not exist between memory images or between a memory image and a present event, but between the original events only, it cannot, in any case, be inspected. There are several issues involved in this objection. We may note first that if there is a direct perception of causal connection, it may be reproduced in memory as well as any other relation or quality. In that case I recall my having perceived the causal relation as I recall my having perceived that one colour pattern was to the right of another.

The problem of inspecting the causal relation of a past

event to a present one does, however, offer difficulties which I cannot here discuss fully. I can only indicate the direction in which I believe the solution lies. That this problem is not peculiar to the causal relation is, I think, clearly indicated by the following questions: How long an interval must follow between two notes before it is no longer possible to know directly and certainly that one is higher than another, supposing a great difference in pitch? Or again, if two colours are successively presented and withdrawn, how long an interval must elapse before we cannot perceive the contrast of two colours? Or again, how many connected propositions can we cover successively in such a way as to include them in a Cartesian 'intuition'? Or again, when does the beginning of an apparently continuous development pass into memory? Approaching the problem from another angle what are the empirical limits of the 'specious present'? Is the 'specious present' measurable by a metronome or by the changes within consciousness? In cases where there are dominant continuities and only slight changes, is the specious present not longer (in terms of measured time) than when changes are fast and radical?

The defining of the limits of the specious present will depend upon the determination of these questions. They suggest that it does not have definite limits. Further, from a strictly empirical point of view, elements which persist without much change or with an orderly development seem, as it were, to be included in the specious present in much larger stretches, so that it is extremely difficult to say when they pass into the category of pure memory. In practice we do not and cannot, for example, distinguish all judgments involving comparison of sense qualities into two classes, one class asserting a relation between two qualities in the specious present which approaches certainty, another class which asserts a relation between a present quality and a past (remembered) one which has, as compared to the first, a large factor of uncertainty due to the fallibility of

memory. Such classes contain extreme cases. There are some judgments which have the certainty of the first class and yet have at least one term of the relation falling outside a strictly defined 'specious present.' So far as such judgments are concerned, the distinction is unreal, or, at most, is a matter of degree. The notion of 'the specious present' though originating from empirical considerations still is largely determined (like the absolute instant) by theoretical ones. Empirically it is difficult to determine what is passing from what has passed and is in immediate memory and what is immediately remembered from what is recalled. On the basis of postulates we may be justified in assuming that these are absolute distinctions, but there is not a correspondingly sharp distinction in the certainty of the judgments which we make about things in and out of this hypothetic unit. Further, any theory which makes this distinction absolute and yet does not postulate an immediate and valid memory is in the way of denying significance to the notion of passing or the past. I think these considerations justify us in disregarding the objection that the causal relation cannot be perceived within 'the specious present' and so cannot be perceived at all.

These *a priori* objections being disposed of, we shall turn now to consider various instances of judgments which assume direct inspection of causal connection. The first of these we have discussed briefly. We have seen that such a judgment as 'I believe this proposition because I have just seen its demonstration' seemed to involve direct perception of causal connection because (1) in the most favorable cases we may be more assured of its truth than of the inductively established law stating regular sequence between 'seeing a demonstration' and 'believing a proposition,' (2) that no reference to a general law seemed to be present in making the judgment, (3) that the confirmation of the judgment takes the direction of an inspection of the circumstances so far as we can reproduce them, so that the meaning of causal connection here

asserted seems to be derived from designating a certain kind of inspectable relation.

We may add by way of argument that it would be very strange if the causal connection in such judgments was not directly perceived since it would follow that no mathematician ever saw any causal connection between his seeing a demonstration and his believing the proposition demonstrated (in the way in which mathematical propositions are 'believed'). Finally, the consequences of assuming that no causal connection between 'having seen evidence for a proposition' and 'believing in it' could ever be seen would prove disastrous to the foundations of any knowledge whatever.

We assume then that mathematicians sometimes know why they believe a proposition, that in the act of deciding a proposition is true, they are sometimes aware of *why* they do so, and I can, of course, only appeal to the reader to ask whether there are not some occasions in which he is aware of at least some of the reasons he has for asserting or believing a proposition. If he is able to find no such instances, he is not likely to find any reasons in this essay for believing in insight into causal connection, and we shall part company with him without insisting that he produce his reason for disbelief in it.

Any judgment may be made on insufficient evidence and we may want to confirm or verify it. The consideration of how we do verify such a judgment as 'I believed (or asserted) this proposition because I had just seen the demonstration' will bring to light the implicit or explicit criteria used in such a verification. The manner of verification depends on the circumstances. If I do not recall the circumstances clearly I regard the case as one falling under a general rule and infer 'it is the sort of proposition I should not have asserted had I not seen the demonstration.' The generalization, i.e., 'commonly I do not assert complex mathematical propositions unless I have seen their demonstrations' may have been reached either partly or altogether by induction and in either case the inference is based on the assumption

of the presence of a *causal law*, and the criteria of this is regularity of sequence. This verification then is made subject to the condition that (1) there is a causal law and (2) this is a case under it.

If, however, the circumstances are vividly remembered we proceed quite differently with the different criteria. We recall the circumstances as clearly and fully as possible with the end of perceiving directly whether or not the understanding of the demonstration was in fact the cause of the belief. I might recall that at the time of asserting the propositions I had in mind, as my reasons for doing so, a memory of just having gone over the demonstration together with a fairly definite idea of how the demonstration went and I might recall that the memory of the demonstration was an essential constituent of the meaning I attached to asserting the proposition. Or again I might recall that I had moved continuously from the demonstration to the conclusion—so there were elements which persisted throughout the whole process of demonstration and assertion.

In either case one asserts that a causal relation was present. But the reason for asserting that the relation is causal and not merely sequential does not lie in any belief or assumption that there is any general correlation between the terms. The events do not have the relation as kinds of events (or as some events or other which have such and such characteristics). One event contains as part of itself a reference to or a memory of another *particular* event. Such a relationship is at least part of what we mean (designatively) by the phrase 'cause of.' We can, I believe, generalize by saying that whenever it is known that one event contains a reference to or memory of an earlier event the earlier event is a causal factor in the later one. This is not a 'causal law' but a definition of what we sometimes mean by 'cause of.' On this definition every memory recognized as such would be recognized to be causally (as well as epistemologically) related to that event of which it was the memory. This I believe would

not be denied, for its denial would mean that it would be possible to have a genuine memory with no knowledge whatever of its causal derivation. We could generalize this still further by saying that whenever we know that one event could not have contained such and such elements but for an earlier event we have a case of causal relation. And this corresponds to what is popularly felt to be the character of the causal relation. The more general definition admits the possibility of causal relations which lie outside mnemonic phenomena. I am not, however, inquiring here whether any such could be known.

The most serious objection to the definition rests upon a sceptical position about memory itself, and this I cannot here discuss fully. On the theory that memory nowhere approaches certainty, that it consists of constructions built upon non-mnemonic data, it would follow that there are no circumstances which could be known to conform to the conditions of the definition. It is, I believe, fair to point out that on such a theory one can find no two events which are *known* to have had a sequential relation to one another and that one could not therefore compile any inductive evidence for 'regular sequence.' This consequence cannot be avoided by supposing that knowledge is concerned only with universals and inductive evidence and assumes only that we know these, for the whole evidential force of inductive evidence consists in knowing that such and such did happen, i.e., it rests on the memory of particular occasions.

Though we have in the judgment which we have been considering a particularly clear instance of the perception of causal connection there are many others which have practically as great certainty, e.g., 'I believe it will rain *because* I read the weather report to that effect,' 'I believe in the theory *because* of the considerable statistical evidence in support of it,' 'I believe he committed the crime *because* he looked guilty.' All these and others like them may in favorable cir-

cumstances be known by inspection to be true beyond reasonable doubt.

Other examples of a somewhat different kind will be found in our accounting for our likes or dislikes. For instance take the judgment 'I dislike him because of an unpleasant remark I heard him make.' Under the most favorable circumstances we may be certain beyond reasonable doubt of the truth of this judgment. If we recall the original circumstances clearly together with the emotion we felt at the time and we now note that the vague memory of that earlier circumstance and its accompanying feeling is a constituent of my present attitude toward the man, we say that my present attitude (roughly designated by dislike) is the effect of having heard him make an unpleasant remark. Such judgments are, of course, subject to error through failure to recall clearly or analyze thoroughly the psychological states involved—but they refer to directly inspectable causal connections and not to interpretations in accordance with inductively established law. And they may in favorable circumstances approach practical certainty.

If we admit more distant and less distinct mnemonic phenomena we may include other instances involving the same kind of causality in a slightly different way. Suppose for instance that I am hearing music which I have heard before, and though the music itself is not particularly depressing I feel depressed while hearing it. I recall that I heard it first in circumstances in which I was very depressed. I now make the judgment that 'this music makes me feel depressed *because* I heard it first under such and such circumstances.' Now there are many causal factors involved in this whole experience but the judgment asserts only one, a causal connection between what I felt in the past and how I feel now. In attempting to confirm this judgment I may argue that my depression could not have been caused by the character of the music itself or other attendant circumstances and that the

laws of association would lead me to expect that my feeling may be attributed to some past circumstances which were probably those I now recall. This, of course, rests on induction and causal law and is probably an element in any well-considered judgment asserting a causal connection in such circumstances. None the less under favorable circumstances we may proceed differently and by a process of developing what seems implicit in our feeling of depression, we recall the state of affairs which, we believe, are uniquely relevant to our feeling. We seem to recognize that this unique feeling (roughly characterized by depression) contains mnemonic elements derived from the earlier experience. In such a development or reconstruction we are of course subject to error. We may, however, reach a very high degree of subjective certainty, in some cases, and we tend to interpret the causal relation in such cases not as an instance of general law but as an instance of the perception that one particular event contained an intrinsic reference or connection to an earlier one and could not have been just what it was but for the earlier event. It should be noted that if the state characterized as depression contained definite memories recognized as such the causal relation would be apparent, and if no definite memories were present only an inductive generalization could be used for verification. But there are cases in which memories are present though not at first attended to and others in which one is uncertain but tends to believe that certain elements are mnemonic but cannot be made definite. This is obviously a matter of degree and we lay stress on general laws in proportion as we are uncertain of the memories.

There is another class of judgments which attribute causal relation as the basis of inspection—namely those which are concerned with purpose or volition as causal factors. Some of these may be reduced to instances of the type we have been considering. For instance in such a judgment as 'I made an effort *because* I had decided to do so and so.' We

have the memory of (or the persistence of) the decision. Such judgments may approach certainty in favorable circumstances. Thus sometimes a chemist may know beyond reasonable doubt that the reason (or cause of) his attempting to set up a certain apparatus is his decision to test the hydrogen ion concentration of a fluid.

The belief that the apparatus has been set up as a *result* of his purpose or decision belongs to a different category, however. It is based in part on induction—namely that certain kinds of effort have been followed by certain physical results; and in part an analogy—the likeness of the set-up of the apparatus to that set-up which he had planned. There is in this later consideration—which is certainly important—an analogy to the intrinsic connection type of causal relation. It differs however since it rests upon likeness. The perceived set-up however does *not contain* in itself reference to or memory of the plan.

I cannot here analyze this type of judgment in detail. I shall state only the general principles which apply. The causal relation between purpose or effort and physical result can be inferred only on the basis of induction; no analysis alone of the effect in comparison to the earlier intention or purpose will give conclusive grounds for asserting the causal relation. There can thus be no instances in which we can say on the basis of inspection alone that the later state could not have happened (or been what it is) except for the earlier.

Our argument may now be summarized as follows: There are judgments asserting causal connections which may attain a certainty greater than the law of regular sequence under which they would be subsumed, that consequently neither the certainty they have, nor their meaning, can be derived from the law. That such judgments are in fact derived from direct inspection of the circumstances, that the principle involved in them is that the effect contains a reference to or memory of the cause as part of itself and that this principle is generalized by saying two events are said to be causally

related when the later could not be what it is except for the earlier. These conditions we found were only met by instances in which memory was involved and so depended upon the existence of instances of valid memory. But a theory of memory which would exclude the kind of knowledge of the past, which would satisfy this definition of causation, would fail also to satisfy the conditions essential to establishing inductive evidence for causal relations.

If we have established the fact that there are judgments asserting causal relations which have a meaning and a verification which cannot be accounted for on the regularity view, and yet which are both consistent with it and presupposed by it, we have not said beyond this, what the relations between them are. In particular it is important to show how purpose as well as memory contributes to supplying an essential characterization to that causally ordered context within which alone correlations of sense data are possible and could possibly have relevance to a knowledge of the physical world. These and related problems I shall leave to some future occasion.

It is, however, to Professor Whitehead's works that the reader must be referred for a comprehensive and profound treatment of these problems and for a metaphysical reconciliation of the two views of causality.

## THE COMPOUND INDIVIDUAL

BY CHARLES HARTSHORNE

*Atoms, cells, and the idea of substance.*—Nearly twenty-four centuries ago appeared perhaps the greatest scientific invention of the Greek mind, the atomic theory. Over a hundred years ago the existence of atoms began to emerge as demonstrated fact. Before this demonstration was completed, empirical evidence had also shown that the atomic principle held sway just where its applicability had been least suspected, in biology. The discovery of cells meant that organic life is atomized, that organisms contain simpler organisms as invisible parts. Today, with all these events long passed into history, their significance seems still largely overlooked, both by scientists and by philosophers. As to the latter, it is almost incredible, but I shall try to show that it is a fact, that not until the twentieth century were philosophical categories developed capable of convenient and fruitful application to an atomic reality, and that anything like a general discussion or even awareness of these categories awaits the future.

Whatever an atom or a cell may be, it is clearly an individual. Of course we are sometimes told that an atom is merely a “construct,” an abstraction, or a convenient way of summarizing certain experimental phenomena. But it is to be objected that, if the atom is a construct or abstraction, it is one of the special kind which posits individuality in the reality which it interprets, and not of the kind which merely refers to a universal, as does the concept “square.” Besides, the cell, at least, cannot easily be regarded, even for a moment, as anything less than a concretely existent individual entity. On the whole the construct theory of atomism is a beautiful example of the widespread attempt of philos-

ophers to adapt the atomic facts to categories which were developed on the assumption that there were no such facts. It is an evasion of the challenging discovery that the variety of individuals in nature is immensely greater than common sense could have guessed, and that all individuals apparent to the senses are compounded of numerous much smaller individuals.

If an atom is an individual, then the philosophic problem of atomism is that of the nature of the individual as such, that is, of whatever is not a universal or mere quality, but is a particular subject or "substance" or existent to which such abstract properties may be ascribed. The word substance is rather unfashionable at present, but if, as there is fair historical warrant for doing, we regard it as simply the technical term for individuality as a philosophical category, then the problem of substance is inescapable. Indeed, the dislike of the word is in part simply an aspect of the modern evasion of the problem of individuality, although it is also a legitimate revulsion from certain outworn attempts to deal with this problem.

*Five theories of substance.*—History shows that five main views have been held concerning the general properties of the individual. Four of these agree in taking literally and absolutely the etymological meaning of "in-dividual"—that is, indivisible, without real parts, simple. The remaining view arose through the discovery that, like so many other doctrines, this one of non-compositeness (to be referred to in this essay as "simplicism," or absolute individualism, or absolute substantialism) occasions difficulties until it is restricted, relativized in some fashion, that is, made a matter of degree rather than of all or nothing.

The notion that the individual has no real parts means that it has no parts which are individuals. Substances can then never contain substances as elements. Substances do not compound to form more complex substances in which the simpler ones retain their substantiality (however this con-

cept is to be defined). Perhaps this veto upon substantial composition has never been rigidly adhered to. But that would of course by no means imply that the objections I have to offer against the non-compositional view are historically irrelevant. For what is needed is a definitive and tenable doctrine concerning composition, not vagueness or inconsistency in adhering to an impossible one.

*Monism.*—Historically the oldest of the four principal forms of simplicism (as a sharply formulated doctrine) appears to be the conception of the entire universe as a single indivisible entity. The first clear-cut metaphysics was the monism of Parmenides. Moreover, monism dominated Indian thought from a very early stage of its development until now. It is also interesting to note that after many centuries of experiment with pluralistic forms of absolute substantialism Europe has seen the potent revival of monism in Spinoza, Bradley, and others. One might seek to explain these facts by suggesting that, in a sense, monism is the most philosophical of simplicist doctrines. For if individuals have no parts then either the universe is the only individual (since others could only be parts of it) or it is not an individual at all. But the latter alternative is little better than a refusal to consider the problem of the cosmic totality; and is not that problem precisely the characteristic and central one for philosophy? On the other hand, monism is highly paradoxical, a fearful defiance both of common sense and of science, and indeed, in itself, a self-contradictory or meaningless conception. So long, therefore, as absolutism rules over theories of substance, one of two things must occur: science and common sense will be discouraged, as in India, or philosophy will be hindered from concentrating upon its chief problems, as in Europe; and in any case the chief philosophical problems will not be solved. In Europe the relation of God to the world was never really faced because the universe, the whole formed by God *and* the world, was scarcely admitted as a topic for discussion; while in India the world was almost given up as a bad

job, as essentially negative or illusory. In Bradley and others the West has also sometimes followed the Oriental example.

It follows from the foregoing considerations that the scientific discovery of the truth of atomism was not, in mere logic, required to show to philosophers the inadequacy of the simplicist view of individuality. For this view cannot deal with the universe, and its incapacity to do so is evident quite apart from the existence of atoms. But even philosophical geniuses are not guided always by strict logic, and it was not until scientific atomism was well entrenched that the hope of finding a tenable version of simplicism began to fade, and the search for a compositional view of substance to begin in earnest. Not that discoveries in natural science were the only cause of the change, for perhaps even more important were developments in pure mathematics, and in aspects of logic related to those developments (particularly the supersession of the "subject-predicate logic" by the logic of relatives).

*Atomism.*—The second oldest version of simplicism is materialistic atomism. For the Greek inventors of this theory, imperceptible particles were the only real existences—"in reality, atoms and the void"—all else, such as human consciousnesses, being mere appearances, mere accidental collocations, or simply gross, superficial views, of the basic reals. The void or space was of course a puzzle, as was consciousness, or appearance, not to mention the totality formed by the atoms and the void and appearances together.

*Aristotelian commonsensism.*—Sharply opposed was the Aristotelian view, which found individual reality in the macroscopic objects apparent to human perception, and denied it to the alleged microscopic constituents. This might be called commonsensism in the theory of substance, for of course common sense expects real things to be of sensible magnitude. Savages believe that there is more in nature than meets the eye of ordinary inspection; but that the real constituents of nature are largely hidden simply because they are so minute is

apparently a speculation occurring only in high cultures and to a few great minds, not to men generally and as such, i.e., as endowed with common sense. Commonsensical in Aristotelianism is also, at least according to a common view of the matter, its dualism, its division of substances into those which do and those which do not possess a soul. But this is not strictly a commonsense view, since primitive peoples seem scarcely to hold it. Also the final causes which Aristotle ascribes to all individuals are difficult to interpret except as traces or germs of soul inherent in even inanimate things. (How else could God influence the world "as the beloved the lover"?) Still there is a real connection between commonsensism, or macroscopic pluralism, and dualism. For if the senses are to be trusted to reveal the individual units of nature, it is not easy to defend the view that all such units are (in principle, or allowing for differences of degree) of one nature. A stone is just not—it seems—an individual in the sense in which a man is so, and if we admit that it is none the less an individual, then the division of individuals into two basically different types is scarcely avoidable. Primitive animism, which vaguely ascribed souls even to stones, breaks down at this point, leaving final causes as puzzling attenuated ghosts of the spirits that preceded them.

It might not appear that Aristotelianism is correctly classifiable as a form of absolute substantialism. Is it not held that the distinction between form and matter is relative, so that what is in itself form may be matter with respect to a higher form, and is not the Aristotelian conception of substance so dependent upon those of form and matter that any relativism applying to these must apply also to substance? Would not Aristotle agree that a brick is a substance relative to clay, and a house relative to the brick, so that the brick is a part of a substance and yet in some degree itself a substance? It is to be observed, however, that any such implications of Aristotelianism are too slightly developed and dubious to constitute a very important factor in the situation when viewed in

the large perspective suitable to our purposes here. If simplification is really untenable, then of course no great philosopher ever fully adopted it; for a philosophical view capable of complete undeviating adoption is not untenable. And on the whole Aristotle made as vigorous an attempt to adopt absolute individualism as has ever been made. Even if he might have agreed that a brick is a substance in a substance, this would not have amounted to relativism in the sense to be set forth presently. The test case of the attitude of the Aristotelians is seen in the doctrine of the human soul. This is defined not simply as "form of the body" but as *the* form, the sole substantial essence, of it. This means that whatever parts, cells, atoms, the body may prove to contain will have no forms of their own and hence no individuality. It is interesting to imagine Aristotle looking through a microscope and beholding the obvious individual forms, dynamic and directive, as well as statically descriptive, of the cells. Would he have remained an Aristotelian? Or would he not have had to confess that his guess had been a bad one? Only Thomas Aquinas, with his theological compulsions, must perhaps have continued to deny cellular individuality in the face of observations to the contrary. Yet either Aristotle or Aquinas would have been quite justified in denying substantiality to cells on their assumption that such substantiality must be absolute if it is to be real at all. Undoubtedly cells are to some extent limited and overshadowed in their independence of action, or in their final causes, by the body as a whole. But there is equally good reason for saying that the soul, as the form of the whole body, is limited by the individual cells. In this way the concept of true reciprocal relativism is reached. But this is to anticipate.

*Berkeleyan subjectivism.*—The fourth and last great theory of substance as absolute is subjectivism, or the view that the only individuals are human minds, or at least, minds not vastly different from the human, with perhaps the exception of a single vastly superior or divine mind. The

Berkeleyan principle that physical things are only ideas resulted from the application of absolute individualism to experience. Things as experienced are parts of the experiencing individual, and this, according to absolutism, excludes their being individuals themselves. Kant only repeats the reasoning, while at the same time grimly insisting that there must be a to us wholly inconceivable remedy for the subjectivist difficulty to which it leads. Things must, he holds, be something in themselves, or apart from us, but it is far from clear how, on Kantian principles, we can affirm this. Hegel also gives no effective solution of the Berkeleyan problem. Many recent realists try to evade it by denying the one sound premiss from which Berkeley had argued, the assertion that things as immediately given are parts of the momentary subject to which they are given. To deny this premiss leads only to the result that the subject is void of content, a whole without real parts, and that there can be nothing which is in any intelligible sense immediately given, at any rate nothing individual. (Cf. *Critical Realism in America*.)

It is noteworthy that subjectivism is a form of commonsensism. Berkeley proudly proclaimed as much, and—except perhaps for the pride—with justice. For basic to his view was the conception of the *minimum sensibile* as the smallest thing that can exist (or rather, subsist as an idea in a mind). The microscopic realm was thus excluded by fiat. Moreover, in rejecting the dualism sometimes regarded as a commonsense doctrine, Berkeley was ridding the common man of an embarrassing encumbrance, since the evidence upon which this dualism rests is invalid if one accepts modern science, with its demonstration that sticks and stones and other dead things are too inadequately revealed to sense perception for the semblance of deadness with which the latter invests them to constitute significant testimony. And the common man in modern times is not ready to defy science. Thus Berkeleyanism is the last stand of common sense in that region in which science shows, if it shows anything, that

common sense is superficial and inaccurate—in very truth a system of “constructs” which are more convenient than literally descriptive of reality. Those who today defend Aristotelianism by calling it “the commonsense philosophy” are simply inviting us to begin the foredoomed process all over again. Every new Aristotle can only usher in a new Berkeley; and both will be in opposition to science.

In order to avoid Berkeley’s error we must first note that his “ideas” were not individuals. This yellow which I now see is not perceptibly different from others which I have seen or may see again. It is *this* yellow for me because it is the one *I now* see. That is, its individuality involves the subjective context. In so far it is indeed my idea. But since, unless some genuine individuality besides my own is given, the epistemological difficulty, the ego-centric predicament, is inescapable, the right conclusion can only be that external objects as known through sense data are not the point of our most immediate contact with reality. That point is within not without the body. Green snakes may, in a case of delirium, be “given” visually when no such snakes are at hand; but never when certain activities in the nervous system are not at hand. The mode of awareness which makes this primacy of the body in givenness almost unmistakably conscious is, as Whitehead well points out, the sense of emotional disturbance. In this sense we are aware of compulsions to undergo certain feelings, and the immediate source of these compulsions we more or less clearly realize to be in our viscera. It is these latter, and not extra-bodily objects, which are then given to us. And the visceral upheavals are really there, unlike the madman’s snakes. Surely the least that can be required of the given is that it should exist. If vision of extra-bodily objects is normally, and within limits, trustworthy it is not because visual objects are givens, but because those states of the nervous system which genuinely are given and guaranteed by visual phenomena are normally produced only when the external objects really are at hand, and this again is due to the

biological adaptation of organism and environment, not to the direct relation of givenness to its given. When, for instance, we see that an object is between two others this betweenness as immediately intuited is a relation between elements in the optical nerve system. The body more or less accurately *duplicates* the pattern of the external object, first in the image on the retina, then in patterns of excitation to and in the brain. This final duplicate pattern is the one we intuit. For the view that the body also closely reproduces something qualitative, like color, in the object we have at present no evidence. Nor is it easy to see what biological purpose would be served, since from qualities no biological utilities seem to follow. On the other hand, since the color is certainly given, and since pattern can in any case not exist without qualities (without terms, no relations) there is no escaping the conclusion that what is given as arranged in patterns must also be given as having quality, and as having the quality which alone is given, that is, the color. If, also, what is given is individual, then one or more individuals, not the subject, must be given as colored; say, as yellow. Admittedly there is no very distinct individuality about the yellow patch as given. But science tells us that nerves have a degree of individuality, and the upshot of the whole matter is, accordingly, that in sense perception the immediate datum is the quality inhering in certain cells, whose separateness from each other is blurred by the non-absolute or imperfect character of the givenness. For here too relativity applies. To intuit any individual whatever with absolute distinctness would be to reach divinity in this respect and, necessarily, all others.

Thus the epistemological problem is one of our relations to individuals, first of all to those individuals with which our direct relations are incomparably the most vivid, the organisms composing the organism we call our body. But Berkeley and most modern philosophers have neglected the individual as such, besides having been strangely obsessed with the extra-

bodily object as the typical terminus of the mind-matter relation, the body being regarded as the mere cause or instrument of this relation.

To say with Peirce that modern philosophy has been mainly nominalistic is not inconsistent with our assertion that it has neglected the individual. For universal and individual are ideas that are clear only in relation to each other, and where either conception is neglected the other will suffer also.

This situation is well illustrated by the brilliant doctrine of American New Realism. This doctrine grants that an entity can be a part of experience and yet real in other contexts as well. But the entities thus "neutral" to experience are not unambiguously individuals, and the wholes which they form will not be clearly distinguished from complex universals. Even if the entire complex of space and time be brought in, still, by virtue of the "externality" of the relations, alternative space-time systems are equally possible, and only the ineffable word "reality" distinguishes the actual from the possible systems. (If the entities were made more or less indefinite, except as related, then the possible worlds would be deficient in the definiteness connoted by actuality. But then, as definite, the entities would have only internal relations.) New Realism has nothing very illuminating to say concerning the ancient problem of universals, and how they differ from and yet qualify individuals. Altogether, a considerable part of recent realist effort to escape from the epistemological tangle into which Berkeley fell has been blocked by the repetition of the mistakes responsible for that fall (neglect of atomism and of the necessity that individuals be given, lack of an adequate theory of the universal). Nevertheless, in destroying the proud and self-confident pseudo-idealism which was based upon the same errors, New Realism and Critical Realism have certainly deserved our gratitude. But the former, at least, abolishes all recognizable individuals, even the human person, which Berkeley makes almost the only true individual.

*The Leibnizian compromise.*—The four forms of absolutism in the theory of substance are now before us. They are: cosmic monism, microscopic pluralism, and two forms of macroscopic pluralism—dualism and subjectivism (neutral pluralism being, as just noted, hardly a theory of substance, or of the individual, at all). None of the four gives a real answer to the inevitable question: how do many things form one universe? Three of the four are contrary to the most obvious findings of science. And the one which alone can accept these findings, microscopic pluralism, is utterly unphilosophical, as Aristotle showed long ago, and for even better reasons than he gave. Above all, like new realism it contradicts the obvious unity and individuality of experience, the epistemological basis for all conceptions of unity. And finally each of the four views has real merits as compared to the others, yet is incompatible with them. Clearly the only hope lies in a new principle by virtue of which these merits can be united in one consistent doctrine, together with the merit lacking to them all of explaining the greatest of existences, the universe itself.

It seems apparent that if atomism is in any sense true, and if macroscopic individuals are also real—as they assuredly are, for we are such individuals—then it cannot be true that individuality is so far absolute as to be exclusive of genuinely individual parts. I say that this seems apparent; yet one great mind expended much of its powers in trying to escape the conclusion, while accepting the premises from which it follows so manifestly. This mind was Leibniz. None ever insisted more than he upon the simplicity of the individual, its lack of real parts, its exclusion of all other individuals. Yet he insisted equally that both microscopic and macroscopic—nay, in some passages, even cosmic—individuality is genuine. He is at once atomist and commonsensist. How is this done? By supplementing the real simplicity of the monad with its ideal or virtual compositeness, connecting the two with the preestablished harmony. No monad actually embraces an-

other; but all monads contain representations of the entire collection of monads. Thus the human soul ideally contains the colony of monads constituting its body, and to a weaker degree those monads constituting its environment. Conversely the bodily units have representations, though of a lower order, of the human soul. Thus on the ideal or virtual side, substantial unity is basically relative, and substances certainly are compounded of each other; while, on the real side, substantial separatedness is absolute. It is a grandiose proposal, but an evasion. For the cosmic unity implied in the preestablished harmony is more than virtual, else there is a regress of harmonizers. The monads do have windows toward God, they do really contain God, since without Him they would be absolutely nothing, creations not yet created. Virtual relativism thus paves the way for true relativism. Leibniz, indeed, came very near to stating the precise logical basis of relativism. The older subject-predicate logic regarded a predicate as that which posits a subject in which it inheres, while a subject posits nothing but itself—"is never a predicate." Leibniz accepted this. But, he asked, what is the relation of inherence of predicate to subject? His answer was the subject is *identical* with the integrated whole of its predicates. In current terms the subject is a *gestalt* and the predicates are partial, abstracted aspects of that *gestalt*. This was a great advance over the old dualism of form and matter. But two problems arise. The first concerns predicates which are relational in character. Each such predicate posits not one but two or more subjects in which it inheres. Yet this predicate which is "in" two or more subjects is itself not two things but one. What becomes of the separateness of subjects thus united? Or, how can one say that such a subject posits only itself? Clearly it posits at least one other subject as term of its relations. The relativity of substance is the legitimate conclusion from this consideration. And Leibniz, on the ideal side of his ambiguous doctrine, fully recognizes this. Each monad internally represents its relations

to all others. We have only to remove the ambiguity, and substantial relativism is reached. It is noteworthy that the resulting compound individual contains as its parts not only those inferior substances, such as cells or atoms, which ordinary science regards as such but also those perhaps equal or superior entities which constitute its neighbors, its environment. These too, though outside, are in some fashion or degree also integral parts of its being. Paradoxically we might say that only some parts of an entity are internal. All this is, after all, only another aspect of the truth which Parmenides and Spinoza set forth, that the cosmos is also an individual in whose unity all distinctions between lesser individuals must be — not abolished, as these thinkers seem to say — but limited, relativized.

The other problem concerns the temporal relations of predicates to each other and to their subjects. Leibniz held that the *gestalt* which is the subject must at all times have the same total content. For, if it were to lose or gain in content, it would no longer be the same substance, but a different one. Identity through time means that past predicates are preserved and future predicates virtually realized already. The older doctrine had distinguished between essential predicates — those defining the identity of the substance — and accidental predicates. The latter constitute change of the subject, but without preventing its endurance as the selfsame subject of these changes. Leibniz, on the other hand, explains endurance too well. He virtually denies change altogether. He falls into the vice of absolutism in demanding that identity be so complete that nothing new can enter the substance. And this absolutism is little more than another aspect of his other absolutist doctrine that actually each substance is simple and outside of every other. For if nothing can enter the substance which is not already a part of its identity, then to explain its adventures no other substance need or can be considered. So Leibniz declares that monads have no windows but yet act in an orderly manner with re-

spect to each other because all are inwardly determined by the initial plan of God. In this way the mutual externality and the absolute determination of the future by the past and present support one another and together prevent Leibniz from discovering the compound individual.

It follows from the foregoing that the distinction between essential and accidental predicates cannot be set aside, as Leibniz proposed, and as every determinist in effect proposes. But on the other hand, Leibniz was brilliantly right in his doctrine of the past. Once a subject has acquired a predicate, it can never absolutely lose it. That which makes the subject the same subject today as yesterday is not, as Aristotelianism taught, merely the enduring common essence, but also the *past accidents*. Reasons for this doctrine are many. First, the past is the realm of facts, and what has been will always have been. In other words, facts about the past are immortal. But what are facts if not propositions whose referents exist? (To say "have existed" begs the question here.) Thus, the immortality of facts means the immortality of events to which the facts refer. The present is a *gestalt* part of whose content is the past. Another reason for this view is that memory is a fact; and memory either dissolves into Hume's paradox of present impressions corresponding to previous impressions even though the very word "previous" is, in terms of impressions, meaningless, or else memory *is* the past as part of the content of the present (subject, like all direct awareness, to erroneous elaboration and inference, to illusions). Here are two reasons, either of which seems to me rather convincing, for Leibniz's great doctrine of the past. The present is a compound individual, of which previous presents are individual parts. And this compounding is, for Leibniz, literal, not virtual merely.

However, in extending his doctrine to the future Leibniz fell into a new version of an error involved in the very doctrine of time he was rejecting. This error is the neglect of the asymmetry of time. For the old view, *neither* past nor

future details were embraced in the *gestalt* of the present substance. (Where this was combined with determinism, the result was an implicit self-contradiction.) This was a symmetrically negative or exclusive view of time. Leibniz adopts a symmetrically positive or inclusive view, as was natural for many reasons. Natural, but unfortunate. For while it is at least highly awkward to deny "once a fact always a fact," it is much more plausible that until an event has occurred its exact character may be undetermined, i.e., not a fact except that, as a matter of fact, it is indefinite. And after all, the future differs somehow from the past. Why not in this way? If the reader will look back at the reasons given for the immortality of the past, he will see that none of them can easily be used to support the pre-existence of the future. Indeed, they are incompatible with it. Corresponding to memory is anticipation; but it differs from memory precisely in the way required by the theory of asymmetry. Memory concerns details (cf. hypnotic recall of long-forgotten particular occurrences, circumstantially reproduced), anticipation concerns generalities, the more distant the more general. And certainly if the arguments did apply symmetrically to past and future we could not speak of the immortality of the past but only of the eternity of all things in a world void of distinction between "past" and "future." If the past is part of what still exists, it must yet be distinguishable from that new part of existence which is the present event. Indeterminism provides for this distinction by making the new part a further determination of an indeterminate feature of the old.

But is the present, then, uncompounded with reference to its future? Does the doctrine of relativity fail at this point? No, for in whatever sense there *is* a future it is embraced in the present. This sense is given empirically in the generalizing character of anticipation already mentioned. The future contains no particulars, but only the law that certain more or less broad or narrow generalities will be *somehow* further

particularized. This law is part of the present, and is the future as such, i.e., as a fact in the present.

There is one feature of Leibniz's doctrine which almost completely anticipates a thoroughgoing relativism. This is his spiritualism or panpsychism. In formal aspect, it is his "law of continuity." All individuals are one in principle, but some are immensely diverse in degree. It becomes only a verbal question whether we say, the least monad is a very low-grade "mind" or "soul," or something different from this only by representing peculiarly low values of the same variables by which minds or souls are compared with one another. Leibniz is the founder of true idealism — not Berkeley or Hegel with their hostility to microscopic and submicroscopic individuals. And the basis of idealism is not epistemological solipsism (the subject aware only of its own states) but the compound individual and the law of continuity, both being aspects of general relativity. The subject is directly aware only of what in this awareness *becomes* part of itself, but from this we must infer just the opposite of the Berkeleyan conclusion that the object is *merely* such a part. Rather we must hold that objects are of such a character that in becoming parts of our individuality they do not cease to be individuals themselves. This can never be if the character of objects is to be dead matter. Thus the basis of idealism is not the ego-centric predicament, interpreted first solipsistically and then, inconsistently, in terms of the absolute mind; but rather the social, the altruistic, nature of immediacy by virtue of which the subject participates in the life of the object, makes this life a part of its own without destroying its characteristic reality. Naturally it could not do so if the object were lifeless. In other words the natures of the subject and its object, while not entirely the same, must differ in degree only, as the law of continuity posits. Using a term of Peirce, this idealism or panpsychism based upon continuity or relativity may be termed "synechistic" idealism.

The demand of realism that knowledge should not alter or

be internal to its object can be readily met by an idealism that respects the asymmetry of time. For if the object is given by memory, then the givenness is indeed extrinsic to the given, since it is related to it as realized detail of its future, which as such, or as in the past, lacked details. If the directly given object is a contemporary of the subject—which Whitehead, perhaps wrongly, denies to be possible—then indeed the awareness must be intrinsic to the object, but this internality is mutual and makes the object no more dependent upon the subject than vice versa. Moreover, although knowing would in this case alter the known from its state previous to the knowledge the alteration might be slight, and in any case the object would be known as it was at the moment of knowing, and further use of this knowledge in memory would be subject to the asymmetry of time which guarantees the past against alteration. Is there any scientific fact which demands more objectivity than this? Of course, scientific knowledge involves a vast quantity of abstraction and inference, and is concerned with laws rather than individuals in their precise individuality, and there is no connection between synechistic idealism and the hypothesis that human inferential knowledge alters the more or less general features of nature with which science is concerned, unless perhaps to some inconceivably minute and insignificant degree, or after some lapse of time so vast that no definite prediction of ours could refer to it.

Leibniz did not invent any one of what we have regarded as the five great theories of substance, because this tolerant politically minded spirit combined several quite incompatible theories, concealing the conflict even from himself under one of the most ingenious intellectual camouflages ever constructed (the preëstablished harmony). But Leibniz certainly came closer to compound or relativistic individualism than any man between Plato and Peirce. About all that is needed is to translate representation into direct awareness, and to recognize the asymmetry of time. Then Leibniz's

great doctrine of perspectives becomes the very key to the cosmos as a one in many and a many in one. For to direct awareness, no less than to representation, must be conceded gradations in vividness, and once this is recognized the importance of inclusion through awareness is seen to be much greater than could otherwise be suspected. If distinct judgmental consciousness, the ability to itemize what one is aware of, is far from coextensive with the range of one's feeling or sheer enjoyment of the environment, then it is not necessary to distinguish between this range and the scope of the environment itself, and the only ground of unity in diversity required for the cosmos is that of dimensions of feeling which participates — in varying systems of gradations of vividness — in other feeling.

*The Platonic anticipation.*— In the *Timaeus* there is a striking anticipation of the doctrine of the compound individual, and an anticipation which refers particularly to the problem which is least satisfactorily dealt with in Leibniz, the problem of the universe as a whole. According to the *Timaeus*, the universe is an organic body which possesses a mind. The grounds for this view are not specified, except that mind is seen as the only self-moved entity, so that if the universe is a dynamic unity it must be a single mind as well as an organized body. But is it not a little strange that Plato failed to remark that the universe is a body which contains bodies (ourselves and the animals we know at least) with the implication that the very principle of individuality is the compounding of organisms into organisms? The conception of a world soul was lost through the obtuseness — if I may be frank — of Aristotle (who, among other blunders, substituted the unmoved prime mover for the cosmic self-mover) and the prejudices, the theological commitments, of the Church theologians, whose lack of disinterestedness was particularly in evidence just here. Thus, when cells were discovered — the one clue which Plato lacked — the philosophical bearings of the discovery were not seen. Not the all-excluding atom but

the organism containing organisms is the model of reality. (The early forms of the "cell theory" somewhat obscured the living, dynamic, organized character of cells, just as physical atomism still obscures the same character in atoms and molecules.) The germs of this view are apparent in stoicism, though weakened by a bias against the atomism of the Epicureans—a true doctrine in so far as it emphasized the compound nature of commonsense objects. But the fear of pantheism was a bar to the free development of compound substantialism, which leads naturally to the pantheistic view of God as the *anima mundi*, or the mind of nature. It is true that Plato distinguished between God and the world soul, but in the light of Whitehead's illuminating doctrine of the primordial and necessary, as contrasted with the "consequent" or contingent, aspects of God it is not hopelessly far-fetched to see in the world soul Plato's account of the consequent aspect, and in his ultimate Creator the primordial aspect. This distinction is one of the conceptions which is required to free pantheism—if the old term can still be used for a doctrine so greatly altered—of its well-known difficulties.

*Whitehead and the compound individual.*—In the "cell theory" or "philosophy of organism" of Whitehead we have nothing less than the first full-blooded, forthright interpretation of the cellular model (passing over the not much less adequate version found in Peirce's theory of the categories, and his doctrine of synechism, both of which conceptions have advantages not entirely paralleled in Whitehead's system). The theory of the enduring individual as a "society" of occasions, interlocked with other such individuals into societies of societies, is the first complete emergence of the compound individual into technical terminology. It is to be emphasized that Whitehead is above all the interpreter of individuality, and of the world of actual individuals as revealed by the inalienable convictions of man and by science. The complexities and obscurities of his writings are partly due to the fact that science shows the actual world to be more

complex than common sense could have dreamt of ; but there is at least one other cause. Being a mathematician as well as natural scientist, Whitehead has, besides his interest in concrete existence, a vivid sense of the reality of universals. This sense is indeed necessary, as we have already suggested, to a clear understanding of individual actuality, and without it Whitehead could not be the great philosopher of actuality that I believe him to be. But the mathematical are only one type of universals, and the study of this type may have led Whitehead to a one-sided view of universals in general. Moreover, he does not seem sufficiently aware of the agonies philosophers have endured, especially in Greece and mediaeval Europe, in searching for a reasonable form of realism in regard to universals. His knowledge of the philosophical tradition is not superficial, but it seems most profound where that tradition deals most explicitly with individuals. In any case I feel very confident that the conception of organism, of societies of entities feeling each other, compounded of each other's feelings, is Whitehead's primary achievement, no matter what is to be thought about his "eternal objects," his theory of non-individual entities. Above all, in articulating a philosophy of substance which follows the contours of existence, distinguishes dimensions and degrees of compounding or societal relationship to fit the facts of inorganic, plant, and animal forms, and in showing how this relational structure resolves the antinomies of subjective and objective, experience and thought, change and permanence, continuity and discreteness, internal relations and freedom, Whitehead has, I have no doubt, achieved the major metaphysical synthesis of our day.

Only a few aspects of this synthesis can be considered in the remainder of the present essay.

*Internal and external parts.*—The human body is a vast nexus or interlocked colony of relatively low-grade individuals, which, in varying degrees, are subject to the control of the human mind. Call the bodily parts a, b, c, etc., and the mind M. Then the relations are these. A, b, c—are

mutually dependent on each other, although certain cells (nerve cells) have greatest influence. (Influence is the joint fact of mutual immanence plus the possession by the influencing unit of creative power, i.e., the awareness of unrealized alternatives compatible with the entity's nature to date.) But M has far more influence over any one of a, b, c, — say n, than the latter has over it (having vastly more creative power). In this sense the bodily parts “belong” to M, namely as its servants. “Internal parts” simply means parts in this service relation, and external parts those not so subordinated to the given entity. The difference is one of degree. Of course, as Hegel said, the master depends upon his slaves as well as vice versa, and even God is no absolute monarch, “without body, parts, or passions,” but is sensitive to, and in a real sense dependent upon, all other individuals. Whitehead would, I think, do well to adopt explicitly the position that nature is God's body, for by the definition just given of a body this is not a mere metaphor.

It may seem contradictory for two entities each to be part of the other. Is not each then less than the other? In a sense yes, for immanence has degrees of vividness or relevance, so that *as in A*, B may be insignificant, and so with A as in B. In-ness, like everything else in a relativist philosophy, is not an all-or-none affair.

In terms of degrees of immanence, degrees of memory, and of originative power, the entire known structure of the world may be interpreted, from space-time as the most general pattern of immanence to the specific characters of photons, molecules, plants, and animals. There is literally nothing like it in current philosophy for richness of detailed implications for science. Quantum and vibratory phenomena are explained as the need of low-grade entities for contrast and repetition within a brief memory span. (See *The Function of Reason*.) The degradation of energy is the gradual loss of zest by such organisms due to their incapacity to originate a new pattern and the fact that the monotonous character of their past (from

a long-range point of view) is not made wholly harmless by the scantness of their memory, since in the dim recesses of faintness memory retains all the past of an entity.

The much-mooted question of whether quantum uncertainty may be utilized by the nervous system to endow the whole organism with freedom acquires a new meaning in terms of the compound individual. For it is really a reversion to simplicist individualism to try to derive all the properties of the body from the laws of the electronic or molecular level. The point is that cells also are units of action, individuals, and likewise the man himself, and that laws, being modes of behavior, habits, of individuals of a given type, must be specific to each type. In the body not merely the arrangement but the inner natures of electrons, atoms, or molecules are different from elsewhere; for their individualities have been in some degree suppressed or modified by the more powerful individuality to which they belong. Thus the body can be free no matter what may be true according to quantum laws; and moreover, it *could* not be free by virtue of the latter *alone*. For if its freedom is merely that of electrons, then as has been well said it is freedom of the electrons but not of the body. This objection to some recent attempts to treat human freedom as simply derivative from quantum mechanics and nerve structure is, I believe, quite valid. It is also to be noted that not merely is the body incompletely determined, even to the extent involved in quantum mechanics, but we must also hold that in so far as the body is subject to laws, these are partly peculiar to the biological level. Both new freedom and new order arise at each level of individuality.

The principal weakness of the foregoing doctrine is that it seems to imply the emergence of new laws, not only between physics and biology, but also between physics and chemistry, or between electrons and atoms, or atoms and molecules. Part of the difficulty of detecting such emergence is that we can hardly experiment upon electrons when they are not to some extent under the control of atomic systems,

and so with atoms and molecules. In any case the earlier forms of emergence theory, which spoke of certain emergent properties as completely inexplicable, are vetoed by the law of continuity. Emergence and freedom are matters of degree, and hence the laws of lower levels apply to some approximation at higher levels. Mechanism as a method in biology is not to be forbidden. Its claims to be the only method, are, however, opposed by most biologists, at least in Europe (cf. K. Goldstein, *Aufbau des Organismus*).

*Compound and composite individuals.*—The neo-idealism of Whitehead, his “reformed subjectivist principle,” is connected with the distinction between colonies which do and colonies which do not involve a dominating (“personal”) unit. Plants and, to a lesser extent, metazoa without brains are of the latter or non-personal class. They are individuals only in a slight degree. We might call them composite instead of compound individuals. Thus “a tree is a democracy.” To ask how it feels is like asking how America feels, except that “America” is in some ways much more unified. The dualism of common sense is due to thinking of composites as more unified and individual than their parts, whereas the reverse is true. A stone is better interpreted as a colony of swirls of atoms (crystals) than are its atoms interpretable as servants or organs of the stone. The atoms and crystals are the substances, the stone-properties, the accidents. But in the animal body, there is truth almost equally in the view of cells as the substantial realities with the whole body as their appearance or functioning, as in the converse view. The animal body and its cells are alive, the stone and its molecules are presumed to be absolutely dead. But this notion of deadness obviously originates from an illegitimate inference from stone-properties to molecule- and atom-properties, whereas the latter are the fundamental ones, by no means directly revealed to the senses which perceive stones. The organism of highly and rhythmically active electrons which is an atom is different from the stone in ways which point in the opposite direction

from materialism. Only aesthetic principles can account for the tendency toward bold contrast, rhythmic repetition, and perpetual process revealed to startled common sense as the real properties of the stupid, inert, merely passive, and non-rhythmic stone. It is just careless to parody this reasoning as amounting to saying that very small units of matter must be less material than large. Size is not involved, but the principles of perpetual change as compared to the possibility of practically complete rest, at least with reference to the near environment (the earth's motion being clearly insignificant to the stone) rhythmic oscillation as compared to mere motion, immanence of the environment and lack of sharp boundaries, as compared to merely discrete units in a void, and, for all we know at least, origination of unpredictable novelty as compared to absolute repetition of a pattern. To be sure, Newton himself was not wholly on the materialistic side of this contrast. Materialism must be a half-hearted doctrine: only a panpsychist can mean literally what he says.

The best mode of attack upon panpsychism known to me is that which may be made by a logical positivist. The question is how far psychological concepts can be handled in science unless physical equivalents are substituted for them. I have too much respect for this issue to try to dispose of it here. I simply point out that it does not bear unambiguously upon the question of dualism in the sense of a division of the world into sentient and insentient individuals. For *all* individuals are merely physical systems for "physicalism." Thus this doctrine apparently accepts the law of continuity. But it is not clear whether or not it consistently adheres to it. Not perhaps without significance too is the tendency to favor a simplicist view of laws as shown in Carnap's remark that the presumption of positivists is in favor of the view that all laws are derivatives from those referring directly to the sub-microscopic level. The societal principle and that of continuity—two aspects of the same thing—can, I suspect, take

care of themselves eventually even against this most vital of all current criticisms.

The non-living societies (of societies of occasions) spoken of by Whitehead are composites too little unified to involve much mutual facilitation between the part-societies. It is not wholly clear to me that a "living but non-personal" society can be other than merely a low degree of the unification involved in the personal type.

*Eternal objects.*—Are eternal objects ingredients in the composition of a compound individual? If so, then what becomes of the unity of the latter? For individuals as ingredients are only relatively distinct from other ingredients; but eternal objects must be absolutely distinct from and independent of any given individuals, since they infinitely antedate the latter. The only way of escape from this antinomy is, I believe, to deny that eternal objects have identical natures as ingredient and as not ingredient in a given individual. How then will they be the same qualities in the two cases? Because before ingression they will be less definite, that is, *more general*. This means that in their eternal aspect they will be completely general, i.e., categories, and that all such specific characters as robin's egg blue are emergents at a certain date, *created* rather than "selected" out of the primordial potentiality. In this way external relatedness will be limited to the relation of future to past. Universals as independent of instances are anticipatory and hence more or less vague. The truth that essences form continua (colors, for example) seems to demand this doctrine, since by Whitehead's own method of extensive abstraction continuity is treated as the possibility of endless division, not as a totality of products of such division. Eternally there is just the unitary vague field of quality, not a set of point-like determinate qualities.

Another advantage of this view is that it avoids the arbitrariness of dividing all entities into sheer individuals, located in space-time, and sheer timeless universals; or into entities

which in their natures completely determine their contexts, and those which leave them completely unspecified. Instead of such a dichotomy, contrary to the general principles of the system, which is based on continuity and relativity, we can set up the principle that independence of context varies through all degrees from zero to the maximum, the former limit being that of individuality, the latter that of pure generality,<sup>1</sup> and the intermediate degrees being the more or less specific traits, such as a certain hue of color, color in general, etc. The more specific and determinate the quality, the nearer it comes to requiring a determinate date of emergence and a determinate distribution in space.<sup>2</sup>

*God.*—The first great problem of metaphysics was, as we have seen, that the universe is a single existent, while it also has as its parts all other existents. The answer to the question, how can this be? is the answer to the question, what do we mean by God? For God is the compound individual who at all times has embraced or will embrace the fullness of all other individuals as existing at those times. He is the only eternal individual, and the only one whose prehensions of others involve impartially complete vividness for all, wherever they may be in space or (past) time.

How do we know that God exists? The universe must have some primordial and everlasting character, as the ultimate subject of change. The past being immortal, there must be a complete cosmic memory, since the past in the present *is* memory. The future being predictable, there must be a world-anticipation; for the future as fact in the present is anticipation. Also, action implies the faith that at no time in the future will it ever be true that it *will* have made no difference whether the action was well-motivated or ill. This condition is met by the affirmation of a God who will never cease to treasure the memory of the action and of its results.

Finally, as an empirical fact, the world is a unified indi-

<sup>1</sup> See my *The Philosophy and Psychology of Sensation*, p. 208.

<sup>2</sup> *Ibid.*, sec. 4.

vidual. We say that a stone is less unified than its atoms (or than its crystals). But in one sense this is untrue. For gravitation, light, and other forces intimately bind the atoms to each other. And we say that this does not unify the stone only because much the same forces bind the stone-atoms to the atoms of air and earth and distant stars—to all the universe. In short, boundaries to the unity of the stone are superficial compared to the unity itself. Otherwise expressed, the unity of the stone as peculiar to it is a rather insignificant modification of the general unity of the cosmos. This means that the latter is much more truly an individual than is the stone. That there is no world-brain can be shown to be a necessary implication of the impartiality of the world-individuality. On a priori or metaphysical grounds, this cosmic unity must be conceded a certain absoluteness of impartiality—the righteousness of God.

Positivists will say that at best we are embroidering upon the strict logical requirements. Memory of the past is more than its mere persistence; world anticipation, more than the mere fact that induction is in principle valid; a righteous world-mind than cosmic order; and so with the rest.<sup>3</sup> But the point is that what "God" adds to these postulates is not mere irrelevant emotional coloring, but the strictly intellectual merit of reducing several first principles to one. Positivism is forever trying to discourage the very search for unity which is science. God is all the first principles as a single principle with an intelligible diversity of aspects. His memory is the past, His plan is the predictable future, His love is His prehension which makes the many individuals one world, His power is the realm of the possible, His enjoyment that of the actual.

This is the first view of God which technically accepts the living, personal, purposive, and therefore temporal, character ascribed to Him by most religions, and which treats the

<sup>3</sup> See Dennes' paper in "College of the Pacific Publications in Philosophy," 1934.

relation of God to man as only a special though supreme case of the relation of any individual to any other, namely the relation of action and reaction, reciprocal inclusion, mutual relevance. God includes us without abolishing our individuality and partial freedom, just as we include cells and electrons. Inclusion being made the basis of all relationship, the idea (which troubled William James and many others) that one mind could include another only by miracle ceases to seem so persuasive.

This is a new cosmology and a new theology, and the symbol of a new era of philosophically enlightened science and religion.

# THE GOOD

BY OTIS H. LEE

## I

THE doctrine that all men by nature desire the good forms the basis for one view of action. In order that this view be made clear, it is necessary that the meaning of the doctrine be determined. Taking this doctrine, then, as the point of departure, our question is as to the nature and function of the good in conduct. We must ask what the good is and, also, what desire is. The discussion, that is to say, will be ethical, in that the good is taken as the end of action rather than as something that is already completely realized. It will also be ontological, for unless the relation of the good to man is entirely accidental, it will be necessary in discussing the good to investigate the nature of man as well, insofar as man acts, and to inquire in what sense the good is constitutive of man as an acting being. In other words, the discussion falls under the heading of metaphysical ethics, as all discussions of ethics must, in the last analysis.

The most formidable opposition to this view is offered by the philosophy which abandons the notion of the good entirely, and maintains that good is nothing but a name for what is desired. This opponent must be dealt with. More common, however, is that philosophy which, fearful of casting itself loose from the notion of the good, relegates it instead to a special sphere of values, where its effect on action either disappears altogether, or is rendered inexplicable. This compromise is an interesting historical curiosity. The realm of values was an invention of the nineteenth century, by which it sought to escape from an estranged world. Unable to live in that world, it sought refuge in another, but the second has

proved as lifeless as the first. Value must be sought in this world, or nowhere, and belief in a sphere of value is only the empty shell of a belief that is already dead. The doctrine that ethics is something separate from metaphysics was a last, desperate effort to save ethics at a time when philosophy had no place for value, and to avoid the embarrassing contradictions which any attempt to bring ethics down to earth would have involved.

Over against the doctrine that men by nature desire the good, we may note two which are at opposite extremes, but are alike in regarding the good as something that is, rather than something in process of realization. For one, the good is a substantive, of which every other entity is a mode or adjective. This conception is mistaken, not because it is too ontological, but because it is bad ontology. If the good is so completely realized that it swallows up everything else, then action ceases to be an important category of ethics, and change, of metaphysics. The good is essentially what is to be realized, and without this character it loses all meaning. It is only a short step from the doctrine that the good is fully realized to the further assertion that the all-encompassing, realized good is neither good nor evil, but simply substance, being, or the one, about which nothing more can be said. This step has been taken more than once in the history of philosophy, and from the predicament of the philosophers who have taken it, a useful lesson may be learned. The fate of a philosophy that will include everything is that it must include its own refutation.

At the opposite extreme stands the logical conception of the good. Good is asserted to be a simple, unanalyzable quality or predicate. True, it is the adjective "good," and not "the good," which is held to be simple and unanalyzable; but the distinction is not significant, for "a good" or "the good" can be only that which is essentially qualified by the adjective "good." It is the adjective which is basic, and the good is merely that on which the adjective happens to descend

— happens, for it is impossible to conceive that an absolutely simple entity could have any essential or intrinsic relation to anything else. This view betrays an unhealthy preoccupation with logic, and complete domination by a naïve way of thinking. There is something radically wrong with the criteria of significant statement, if they can forbid the assertion that good is pleasure, just as they forbid the assertion that black is white, and yet fail to distinguish deeply between propositions so different in type as even “This apple is red” and “This apple is good.” The fundamental error, of course, is the initial assumption that good is an adjective. The assumption is perfectly gratuitous, and not the slightest evidence or argument has been offered for it. If it is granted, everything else follows; but why it should be granted, or how it can be believed, is difficult to see. When it is asserted that all men desire the good, the meaning is not that all men desire that which is always qualified by a certain simple, unanalyzable predicate. On this theory, moreover, the bond between good and existence has been broken, for it cannot be shown that the simple quality requires realization since, being simple, it is self-sufficient. Nor can any reason be given why one should desire things which are good rather than things which are yellow or sweet or hard. This last, however, is a consequence which any holder of the view will not only admit but insist upon, if he understands the implications of his position. How much of ethics then remains standing had perhaps best be left unsaid.

The position is closely allied to hedonism, in spite of the absurdities of which it accuses the hedonists. In fact, it developed out of a long hedonistic tradition. The development was possible, almost natural, since both hold the good to be adjectival in character. But in another perspective the development was unnatural and hence carries little conviction, for the tradition was nominalistic as well as hedonistic; and the logical predicate “good” in an atmosphere instinctively hostile to universals was an anomaly. It could not make itself

at home, or find a part to play. The simplest remedy would be to add that the predicate is only a word. Then all would be well again, and the tradition untroubled by strange innovations. The difference between the two views is that hedonism still remains within the universe of ethical discourse, since pleasure is a predicate only to be found in connection with a sensitive, desiring subject; while good is a predicate which has severed all connections. But the reply to both is the same: pleasure and the good, or what is pleasant and what is good, may be found among the objects of desire, but they can claim no preeminent or exclusive place there.

So much by way of introduction. The good is not something completed, whether that something be adjective or substantive; it is always in process of realization. The good is individuality, not your individuality or mine, but individuality as such; and in desiring the good, what men desire is always individuality. The best approach to these theses is through a consideration of the nature of desire.

## II

THE good as a philosophical category has almost disappeared from contemporary discussion. Instead, action is treated from the point of view of interests; and so far as philosophy is naturalistic, as it is to a very great extent nowadays, interests are conceived mostly in physiological terms. This point of view adheres to the principle that we do not desire things because they are good, but, on the contrary, we call things good because we desire them. The precise sense in which they actually are good through being desired may be difficult to determine, but this is a minor question, in comparison with the determining principle. It is the contention of this essay that we desire things because they are good, and not conversely. The whole question turns on the nature of desire.

The Greeks were puzzled by the problem of learning. We cannot inquire into that which we know, they said, since we already know it; and we cannot inquire into that which

we do not know, for we do not know what that is; nor, if by chance we come upon that into which we were supposed to be inquiring, shall we be able to identify it as the object of our previous inquiry. This is a special case of the problem of all becoming, for becoming involves continuity. Unless an identity is maintained through a process, nothing has become. In this case it is the mind which at one time is to become, and at a later time has become learned. But if the dilemma holds, that becoming which is learning is impossible, and the mind is only a theatre where ideas appear and disappear again. For unless ideas can be sought and found, the mind must at every moment be identified with its actual, explicit contents. On this view there can be no connection between what is actually present in consciousness and what is latent or potential. The mind will be all actuality, to the exclusion of potentiality; it will exist in the present, to the exclusion of past and future; it will be a stream of instantaneous states.

Desire involves an exactly analogous dilemma. We cannot desire what we have, since we already have it; and we cannot desire what we do not have, because if we do not have it, we cannot know what we are desiring; and if we attain it, we shall never know that our satisfaction is the satisfaction of the previous desire.

Now it will be readily — too readily — admitted that we do not desire what we have, but there will be a cry that the second half of the dilemma is sophistical. Obviously, it will be said, we cannot desire what we in no sense have, but we do have the object of desire in a certain sense, for we have an *idea* of it. What is the source of that idea? It comes from past experience. I see an orange growing on a tree. Having eaten oranges in the past, and found them good, I pick this one and eat it, recognizing it to be an orange from its similarity to the ones I ate before. And yet, this answer is not satisfactory, for in the first place I do not desire the oranges I ate in the past, but this one; yet the idea which I have from past

experience is either the idea of the past oranges or the general idea of orange, associated, let us say, with an idea of pleasure or satisfaction. It is not the idea of *this* orange. Moreover, it may be that I never before ate an orange, and hence, although one now presents itself to me, there is nothing in past experience which leads me to want it. One may say, of course, that I eat it because other people do so, and I am ashamed to reveal my lack of experience before them; but this is simply to fall back again on past experience, for I now desire social approbation, something with which I have long been familiar, and not the orange after all. Unless one is willing to bring forward an instinct of curiosity, there is only one answer left, and that is that we come upon new types of satisfaction by chance and random activity, which types of satisfaction then, in turn, give rise to new objects of desire. In that case, all desire is to be accounted for in terms of chance and past experience, since no one who has gone this far, it may be assumed, will assert that there are innate ideas of objects of desire.

So far as action is purposeful, then, it is to be accounted for in terms of past experience. The solution is inadequate. Past experience does play a part, but it never can account entirely for any desire; for the fact remains that I do not desire either what I experienced in the past, or the general idea, but a particular future satisfaction, and past experience cannot account for this. The empirical account is inadequate. To the question, whether we desire what we have, or what we do not have, it answers with an evasion: we desire what we had at some time in the past, or rather, something like it. But the dilemma must be solved, if at all, in its radical form.

We desire both what we have and what we do not have. The object of desire is individuality, and the desiring subject is already an individual. Philosophers have said that substance does not admit of degree, but they were mistaken, for an individual can possess a greater or less degree of individuality. The individual already possesses individuality, but desires

more, and the satisfaction of desire is always, to some extent, a realization of individuality — which does not mean, of course, that it may not also involve a loss of individuality in another direction. In satisfaction, the individual becomes what it already is. The dilemma of desire, like that of learning, declares that the object is either completely diverse from the subject, or completely one with it. The reply is, that a complete diversity does not exist, because subject and object have in common individuality; it is the individuality of the desiring individual which is realized in the satisfaction of desire. Yet the two are not one, for in desire the desired individuality is not yet attained. The principle of individuality will not, alone, explain why desire centers on this or that particular object, but it does supply the explanation of the possibility of desire, and its essential nature, because it bridges the gap between subject and object.

If this were not so, there might be satisfactions, but there never could be satisfactions of desire; for that which satisfies is always something which, strictly speaking, has not satisfied before. The desire for it, therefore, cannot be accounted for in terms of what has satisfied. Nor can it be accounted for simply as a present experience, for the question is not, how we can be satisfied, but how our desire can be satisfied; and there is no explanation, unless seeing is desiring, of how there can be desire at all. Besides, desire is often of something that is not given in present sensuous experience.

There is always present in desire a second factor, in addition to individuality, and that is the idea. The idea of the object may be more or less determinate. Furthermore, it is of the nature of a description, which may be either definite or indefinite. The object may be simply "a walk," quite indeterminate; not a walk of any specific character, but simply a walk. On the other hand, it may be a walk of a certain duration, in pleasant country, on a cool day. Such a desire would not be satisfied by an hour's exercise on a treadmill. Yet, no matter how specific the description be

made, it remains indefinite, and would be satisfied by any object which happened to fit it. Some object, or any object of the description "x" is desired, but not this particular object: though the object is always particular, it is not always a designated particular. It is not always a "this," or even a "that," removed in time and space. Yet individuality enters into such a desire, the object of which is the desiring individual participating in a process and performing an action of a certain character. But further, the object, in abstraction from the intended action of the desiring subject, may also be definite. It may be a walk with a certain companion which is desired, and the companion may be either the person with whom I walked yesterday, or *this* person, present here and now. Generally speaking, when little of our own individuality is involved, we do not demand individuality in the object. We are satisfied with "a walk," or "a drink." But when our own individuality is greatly involved, we demand other individuals; we wish to walk with a certain person and no other, or to enter into a particular social activity with specified friends.

Two extreme cases are to be noted. The individual may experience a general restlessness due to the extreme indeterminateness of the idea involved in his desire. He can assign no specific cause for his restlessness, which is directed toward no discoverable object. He is impelled to act in any way whatever that occurs to him. We scarcely can speak of a desire in such a case. One factor, namely the individual, has almost excluded the other, that is, the idea. There is also another sort of restlessness, which is a dissatisfaction similar to the first on the surface, but opposite in nature. It is a state in which nothing will satisfy or please. But here the idea is too definite and dominant. The desire is a nostalgia, a longing for something experienced in the past, which cannot be recovered; and because the past action cannot be reenacted, nothing, no matter how carefully chosen, will satisfy. Desire oscillates between these two limits, according as the element derived

from past experience is greater or less. The absolute extremes, however, are never reached: idea is never entirely absent; nor can the past, just as it was, be desired. Intervening experience has changed the individual and hence his desires; he wants, not a concrete past, but certain aspects of it.

But in any case, the ideas due to past experience, and imagination based on these materials are not sufficient. Put in traditional terms, desire involves an innate element. Yet this element is as truly empirical as any other, for the experience of individuality is original and continuous, present at every moment.

So far, desire has been considered in relation to a single individual. We must now deal with its relation to individuality generally. The good is individuality as such, and not the individuality of a particular individual, it was said. The reason is, that the existence and realization of one individual are bound up with the realization of others. Individuality is a social affair, and one individual in isolation is an impossibility. Let this be granted for the moment. Then the realization of many individuals is objectively best — but does any individual actually desire the realization of individuality other than its own? Unless this can be shown, there is no justification for the assertion that men desire the good. The utilitarians could not explain the possibility of such desire, and this failure wrecked their philosophy. Utilitarianism was a social philosophy, founded on a conception of the individual which excluded every social act and motive. Though the good was recognized to be social, therefore, it could not be explained how anyone could pursue it.

The possibility of social acts rests on the social nature of the individual. Because the experience of an individual consists, in large part, of the experience of society or societies — that is to say, of other individuals in various relationships — action is social in character. It has social ends. The nature of man is largely determined by the society in which he lives, so that society is the cause of his character to that

extent. So much is universally recognized. But we must go a step beyond this. The character derived from society is social not only in that it is a product of society, but also in that it contains social attitudes. The relation between the two is dynamic; what the individual does as well as what he is shows his derivation. If the aims of society are selfish, his aims will be largely selfish; but if the aims of society are social, his will also be social. A social act is one performed for the sake of all the individuality involved in that act, and the individual who participates in such an act enters into it. It also enters into him. He wishes not only to make it part of his experience—or to derive pleasure from it, as the utilitarians would say—but also to enter into the experience of others. In entering into the activity, he is seeking to realize not only his own individuality, but that of the others as well. A man does not desire merely that others should become parts of himself. He does not attempt to absorb others into his own experience; he does not attempt to become an absolute in any other sense than, being an individual, he already is. He desires that others shall become parts of his experience, but he also desires to become a part of their experience, so that society cannot be regarded always as a mere means to the satisfaction of individual ends.

Desire, then, is largely for participation. The most obvious case where it does not have this social character is the process of nutrition. There the object either is not individual or consists of individuals with which we cannot communicate. This type of desire is destructive of its object, and includes all those desires which are usually called the purely physical. In the sphere of social relations, the same destructive tendency appears as the domination of one individual over others. It may take the form of open slavery, or it may be more refined, but in any case it involves the use or threat of force as such. Private property, where it is not greatly restricted, is one of the commonest forms of such domination. Pure physical desire in this sense is very rare in human activity,

if indeed it exists at all, for nearly all of our desires clearly have a social reference. Eating, for example, normally involves private property. If there is any merely physical desire, it does not fall within the scope of ethics, for the field of ethics is society. Desire has been conceived far too much in terms of destruction. Most of our desires, in friendship, business, political life, education, sport and religion, are not of this sort. They are wholly or in part social, and their fulfillment is a cooperative activity in which each enters into the experience of the others.

The tendency to interpret desire in physical terms is interesting. It in large part explains the widespread condemnation of desire on the part of moralists. Perceiving the destructive character of physical desires, and interpreting all other desire after this pattern, they naturally condemned desire as immoral. They were right, granting the interpretation; they were even right about physical desire, so far as it has undesirable social implications.

The same tendency is largely responsible for the older egoistic systems of ethics. The individual, not being conceived in social terms, was thought to have both desires and their satisfactions within himself. It is no accident that, in the catalogues of passions, desires, and pleasures and pains to be found in the works of seventeenth and eighteenth century writers, those which are either physical or "self-regarding" so often predominate. Bentham, for example, near the end of the period, says that the only pleasures and pains (in a fairly long list) which are extra-regarding are those of benevolence and malevolence. On this theory, desire is caused either by the activity of an object on a passive subject, or by something going on within the subject itself. The process of satisfaction is a causal activity on the part of the subject, directed toward the object. The result, if successful, is pleasure, an adjective of the subject. The sole relation between individuals is causal. The causal theory of perception

led to epistemological difficulties; the causal theory of desire is simply its ethical corollary, and resulted inevitably in egoism, which is the ethical equivalent of solipsism.

The categories of causality and means and end are inadequate to express the facts of human action. The inadequacy of the category of means to an end was seen by Kant, who dismissed it from ethics under the name of the hypothetical imperative. Certainly efficiency, defined as the understanding of the most effective and economical means to any given end, can solve no problems of action. Yet this conception of means and end is tremendously significant. It rests on the premise that reason is concerned with means, while desire determines ends. Hume asserted the premise when he declared that reason is, and ought only to be, the slave of the passions. Kant, whose position is of course very close to that of Hume at the decisive points, could escape the same conclusion only by a doctrine that split his philosophy in two. Even so, the practical reason only enabled him to say that reason ought not to be the slave of the passions, not that in fact it is not so.

The separation of reason and desire is the most vicious doctrine in all modern philosophy. If it were true, rational action would be impossible. Yet it is only the logical conclusion from the doctrine that desire is purely physical. It appears in philosophy in many forms. One is the theory of emotive meaning and the emotive use of language. Since other kinds of meaning have no motive force, it is necessary to discover a special kind which does have power to produce action. Another form of the doctrine is the theory that philosophy is method, that it has to do with the discovery of means to given ends, and not with the ends themselves. This is merely an inversion of the position of Kant. For him, the question of means was a technical rather than a moral problem; for instrumentalism, the problem of means is everything, ends being already given.

Now the question of how the end is to be attained is of

first importance, if the end of action is individuality; for the end is something to be realized, not contemplated, or even striven for in vain. Therefore, the problem of means is highly relevant to ethical discourse. But the determination of ends requires reason, and the determination of means involves desire. The two cannot be separated in ethics without fatal consequences. The satisfaction of certain physical desires and needs, for example, is a condition of or a means to individuality. It has nothing to do with ethics, however, unless social questions are involved: not because the food supply fails, is there an issue, but only because someone deprives someone else of the means of subsistence, or has been negligent in allowing the supply to fail. Socially, the separation of means and end is impossible because it destroys the social character of experience. The individual who is only a means does not belong to the society which uses him, nor does the individual who is only an end belong to the society which he uses to attain his personal objects.

Efficiency has another meaning. The efficient individual is one who is capable of meeting any situation that arises; and the good man is the efficient man in action. But efficiency is more than the ability to reach a given end; it is also the insight which apprehends the specific end called for by the situation. It is categorical, not hypothetical: it combines reason and desire, and the result is good and effective action.

The selfish individual is one who does not see that his individuality requires that of others. He desires individuality, but can recognize only one individual, namely himself. He is lacking in understanding of his own nature, and his actions reflect his ignorance. In this sense, every individual is more or less selfish, for the experience or intention of no individual extends to all others. It is a matter of degree. Antisocial activity is destructive in character. The man, for example, who seeks to dominate others is performing a social act, for his acts are directed at his fellow men. Yet in dominating

them, he is reducing them toward the level of the food he eats. His domination destroys the social relationship between him and them, for social relationship is reciprocal in character. The more domination approaches the type of desire for nutrition, the less social it becomes. The extreme is the pure relationship of means and end, where all sociality has disappeared.

## III

ALTHOUGH the social character of individuality has been nominally recognized in modern philosophy now and again, it has never formed an integral part of modern philosophical thought. All the important principles of both the British and the German traditions have excluded such a doctrine. The best of German thought, the period from Kant to Hegel, tended increasingly to regard individuals as modifications of a substance. The British tradition, on the other hand, under the influence of the old system of classification by genus, species and instance, viewed individuals as instances of a class. Having lost the Aristotelian metaphysics underlying this scheme, it lost at the same time the insight that man is a political animal, and could conceive of no more fundamental relations between men than those dictated by self-interest or a compact. British thought lost the one in the many, while German thought lost the many in the one.

Kant attempted to give the social nature of individuality an ethical expression in the ideal of a kingdom of ends, while the utilitarians put forward as their criterion the greatest happiness of the greatest number. But neither Kant nor the utilitarians could support their ethical convictions by a metaphysic that was compatible with them, and so both, though in somewhat different ways, fell into difficulties.

The notion of a kingdom of ends is the most profound insight of modern philosophy into the social nature of experience and the ideal of action. Its development by Kant, however, was vitiated by undue emphasis on the motive, and

neglect, though not denial, of the significance of the action to which the good will gives rise. It was haunted by the dualism which separated transcendental choice and empirical act; since the act had to be conceived as at least partly empirical, the metaphysic was fatal to the ethical doctrine. The formalism of "consciousness in general" gave rise to the philosophy of absolute spirit; and with the attainment of absolute spirit German philosophy lost the insight that one individual cannot exist alone, since by absorbing its other, it destroys itself.

The utilitarian formula, if a more homely conception, gave promise at the same time of being an ideal which might have an immediate and specific bearing on the actions of men. Practically speaking, it did work for a time. Yet the tradition behind the utilitarians made it impossible for them to explain how anyone could pursue the ideal, since everyone necessarily acted in order to attain his own greatest pleasure. They could find no way out but to postulate a miraculous harmony between virtue and self-interest, between the social and the individual good. From a practical point of view, since everything was due to association, it was doubtless possible, by tradition and education, to establish associations between actual or anticipated pleasures on the one hand, and certain ideas on the other, in such a way that socially desirable ends should be realized. We may pass over the questions, whether the distinction between ideas and feelings can be maintained; if it cannot, whether feelings and actions can be explained at all; and if it can, whether entities so diverse as ideas and feelings can be associated by principles which were asserted to unite ideas. But granting satisfactory answers, such interference could be justified only on the basis of a metaphysical theory of the real, but as yet undeveloped nature of the individuals whose lives were being interfered with; and such a theory the empirical utilitarians did not have. Persons who attempted reforms were simply striving, like everyone else, after the greatest available lot of pleasure for themselves.

Why not after a social goal as well? Because for this tradition there could be no such thing as a common object, either of knowledge or of action. The object of action was the attainment of pleasurable feelings which admittedly belonged to the private experience of the individual. The objects of knowledge, ideas, were no less private and subjective; but even if they had not been so, the theory that reason is theoretical, that it deals with means only, and never with ends, the latter being determined by feelings, sentiments and desires — this theory alone would have ruled out the possibility of social ends of action. Neither the existence of social ends, nor the possibility of their being pursued by anyone, admitted of an explanation. In fact, the separation of reason and desire, of ideas and feelings, made rational action, whether social *or* selfish in intent, impossible. Instinctively social actions through the sentiments of sympathy and benevolence were not ruled out, granting a common world, but these could hardly be called ethical. Besides, the belief in such a miraculous harmony as they implied rested on an optimism which could not last long. Since the good of the individual was stated independently of society, it could be but the sheerest accident if the goods of individual and society happened to coincide. No wonder that some people had doubts concerning the reality of the coincidence. But whether it existed or not, it could make no difference to the actions of the individual, who would go on, in any case, using society as a means to his own ends.

Historically, the individualism of British thought goes back to the subjectivism of Descartes; for if the experience of the individual, or better, the individual experience of the individual, is the primary datum, then sociality must be a derivative character. The experience of the other *as other*, the most obvious and primitive experience of all, must be interpreted to fit the experience of the individual as itself. Having started on this road, British thought could not escape, being determined in its development by modern science and the

epistemological problem which it set. German thought was less historically determined. One might interpret it, too, in terms of the epistemological problem and its solution through the identification of finite with absolute reason, but such an interpretation would be superficial. The epistemological problem was here always a factor of secondary importance. Through and through, German idealism was the philosophical expression of poetry and religious mysticism. The one tradition was scientific, the other religious, in spirit.

Both of the traditional ways of conceiving the relations between individuals are inadequate. The mutual dependence of individuals can be expressed neither in terms of common membership in a class, nor of common modal dependence on a substance. Individuals are essentially social; the being of one implies the being of others, and one can exist only through others. Put in traditional terms, which however restrict the notion of individuality to a special case, the self exists only through the not-self and through other selves. I become aware of my individuality only when I distinguish myself from my fellow men. This is a commonplace. But further, my existence, as well as my awareness of it, depends on my relations to others.

This interdependence can be expressed in many ways. It can be stated in psychological, or sociological, or physiological terms. In philosophy it must be expressed as the metaphysical dialectic of the one and the many, as the truth that the one exists only through the many, and the many through each other. The individual participates in other individuals, and this participation takes the forms both of knowledge and of action. Without the objectivity of common knowledge, without the direction of action toward and the expression of functions in terms of others, the single individual would have no existence.

The good likewise must be conceived in terms of the mutual participation of coordinate individuals. That individuals require one another was recognized implicitly in the

history of philosophy by the principle of the plenitude of being, according to which divine perfection requires that every degree of possible reality be exemplified in existence. The principle also suggests that the good is the maximum of individuality, although the individuality of finite beings is considered instrumental to that of the infinite being. The inadequacy of the principle lies in its portrayal of the relations between individuals in terms of a vertical, linear hierarchy, in which the place of each is determined by its degree of reality, measured against the ideal of infinite being. But the fundamental relation of participation, although it involves subordination, must be understood primarily in terms of coordination rather than subordination.

## IV

PHILOSOPHIES built on categories which admit of degrees run the danger of falling into romanticism. Romanticism is the inability to see the limits between different types of entities, and to make explicit the various forms in which a principle finds its exemplification. Without the patient and careful elaboration of such forms, the principle must remain fruitless, and the philosophy which scorns this prosaic inquiry is only a warm feeling of elevation. The intervening steps which connect the generality of the principle and the concrete particularity of its exemplifications make up the content of a developed philosophy. For this reason, individuality as the principle of value does not supply a criterion for action. It cannot do so until the various forms which the relations between individuals may take have been set forth in detail, so that it can be seen which ones are productive of a high degree of individuality and which are not. This task cannot be undertaken here. Certain directions that are to be followed out, and others that must be avoided, can be indicated, however.

First, what is individuality? It is indefinable. Definition

involves concepts, but there is not a concept of individuality, in the sense in which there is a concept of tree or of causality. The individuality of an individual is not like the greenness of a leaf, or even the humanity of a man. It involves uniqueness, yet is not synonymous with it; for greenness is unique, yet greenness does not possess individuality. It is possessed only by experiencing subjects, yet it is not synonymous with experience; for to say that the good is individuality is not to say that the good is experience. Although experience is good, possesses value, it is not the good. Individuality is not a quality which can be predicated of a subject. Of course, we do say that this or that person has individuality, and we may mean to designate something definable by the expression, such as vivacity, or wit, or eccentricity; but this is not the individuality in question. We are more likely, if pressed, to answer that individuality is something, we know not what.

It might be asked, in reply to the assertion that the good is individuality, whether this means that each individual has its own criterion of the good, or whether it means that there is a single standard which holds universally. The question is misleading, for it is framed in terms of the controversy between nominalism and realism. Thus the choice appears to lie between admitting that the good is different for every individual — i.e., that there is no such thing as the good, since the good is simply what is desired — and holding that the good is a concept or universal law. The attempt to escape this dilemma led to the theory of the concrete universal. The answer is, that individuality does not belong to one individual in isolation: the question implies the opposite. Individuals in relation to one another have, through their interactions with and participations in one another, different natures, which yet are not entirely diverse, or independent of one another. The opposition between universal and particular is not relevant to the problem. The notions of identity and difference, so far as they are conceived in terms of universal and particular, are

equally irrelevant. Essential participation is an idea foreign to the popular philosophical conception of either particulars or universals.

In the second place, there cannot be two criteria of action, one for the individual and another for society. It is impossible to make a distinction between individual and social ethics without ruining ethics. There is an element of privacy in experience, to be sure, but it belongs to religion not to ethics. From the standpoint of ethics, private experience is either that which is merely subjective, that which has been found not to have an objective reference, or it is that which is potentially social. The artist and the scientist, the statesman and the prophet, are individually creative, but their genius is social in its consequences. Furthermore, a distinction between individual and social ethics cannot be defended on the ground that the former deals with motives, the latter with the overt part of actions and with their consequences. Motives are very relevant in ethics, perhaps most important of all, but not apart from their consequences; on the contrary, they are relevant just because of their relation to the actions in which they eventuate. Motives are to be evaluated pragmatically in every case, not in themselves, but in relation to the consequences to which they lead. We do recognize that a motive may not gain expression in action, and rightly so—"with intent to injure" or "without intent to injure"—but the motive is taken always as an indication of a certain character which, if it is not effective now, will nevertheless eventually make a difference socially.

The good is the socialized individual or the individualized society; but the duality of statement should not convey a duality of significance. The first phrase does not mean that society is a means to the development of the individual; the second does not mean that the members of a society are only its modes. The former gives us the social contract; the latter, the absolutist conception of the state. Neither of these has

a place for the sort of reciprocity which is the essence of all social relationships.

It was the duality in its conception of value which, above all, wrecked utilitarianism. Many criticisms can be made of ethical hedonism, but the one which is fatal is implied in utilitarianism itself from beginning to end. The existence of "extra-regarding" actions can be explained only through the social character of the individual who acts; without this, the inevitable outcome is a split between individual and social ends which no amount of patching can hide.

The philosophy of organism bears traces of the same duality, for the value of an occasion of experience is stated in terms of an intense, harmonious pattern of feeling, a formulation which implies such privacy and isolation of the occasion as exclude all reference beyond it. The reason is, that value is a matter of the present, and in the present the occasion is alone. It can never experience the other as other, but only as in itself, since when the other is experienced it has already passed away as occasion. This interpretation must be modified by the addition of two doctrines, the conformity of feeling to the past, and the sense of importance directed toward the future, which express in terms of value the relation of the occasion to the cosmos; when they diminish, experience becomes trivial. Nevertheless, beginning in this way, one cannot arrive at a satisfactory conception of the values of society, for the aesthetic criterion is an individual criterion. It does not really explain why, from the point of view of value, relation to the cosmos is essential.

The most perfect expression of the social nature of value is to be found in the *Republic*. There Plato points out that justice is the same in the state and in the citizen. The state is founded on the human characters of its citizens; and conversely, the characters and functions of the different citizens are based on the nature of the state. In the ideal society there is complete harmony between the two, nor can we say that one is more fundamental than the other.

And yet, Plato does say exactly this, for he adds that justice is concerned with the inward man. The philosopher cannot accomplish his greatest work unless he finds a state that is suitable to him, but his primary concern is "the city which is within him," where he will always live, whether the other ever exists or not. This is the truth, and the philosophy which pictures a perfect balance between opposites is only a compromise. In what, then, does the priority of the individual over society consist? It consists in the fact that the actions and events which are the life of society are the choices of individuals.

Thirdly, therefore, we may distinguish in social experience the two aspects of structure and function; and these are two aspects of individuality, of which the second takes first place. The structure of the social world in which we live is itself social. The various forms—political, economic, domestic, and so on—which define what that world actually *is*, are the products of society; and society includes, naturally, not only the other members, but oneself as well. These forms are the deposits of past experience, preserved in customs, institutions and laws. Structure is both intensive and extensive; that is, it may vary in its elements, articulation and integration on the one hand, and in its extent, according to the society, on the other. Every society exhibits these two sides in its structure. The functions in society, on the other hand, are all carried on by individuals. They are social, in that they have reference to other members of society, and embody social structures, but they are the activities of individuals. The priority of the individual over society is the priority of the social process over the structure of society.

When experience is interpreted in terms of aesthetics, and value is described as a harmonious pattern of feelings, the reference is primarily to structure, for it is the pattern of the experienced world which is chiefly intended. But the terminology of aesthetics is either erroneous, or so strained as to be inapplicable, for the obvious aesthetic fact is completeness,

while experience is ever incomplete. Also, the next step in this direction is the doctrine that the good is the greatest number of aesthetically vivid experiences, a position too close to utilitarianism and the hedonistic calculus to avoid the accompanying difficulties. Given any pattern of experience, realized in a moment, a day, a lifetime — can it be regarded as complete? It never can be, for the sociality of that experience is incomplete. The sense of importance is the recognition of incompleteness, but the addition makes all the difference, for it implies the understanding of value in terms of society instead of art. But must not society itself be evaluated according to aesthetic standards? This cannot be granted, for society is the wider category, and art, even in the broadest sense, is not the only social activity or attitude.

The function of the individual is, simply stated, choice and action. Every individual strives for individuality, in his own way, and according as his understanding of individuality is more or less comprehensive of others. It is by such choices and actions that society lives, for they actualize the functions which every individual embodies. It is obvious that all action is social in one respect: from a certain point of view, the criminal acts as socially as anyone else. Why, then, does the rest of society call him antisocial and take action against him? It is only because all men desire the same good that there is a basis for ethical discourse and a justification of action on the part of society. Without the common end, mutual understanding would be impossible, for what was good to one would be evil to the other, and what was wisdom to one, would be stupidity to the other. It furnishes also the only justification of social action toward any given individual, for unless that individual does seek the good, interference with his actions is the exercise of bare force; if he does, such interference takes the form of education. But if everyone desires the good, actions must be determined by knowledge. In that case, can anyone be said to make choices at all? Certainly. To suppose that because choice is determined by knowledge

the individual has no choice, is to misunderstand the nature of thinking, which cannot be separated from action. This is clearly a circle, and the problem it involves must be followed to the last possible limit; but it is a circle from which, in the last analysis, the only complete escape is fatal to ethics. To avoid it would be to return to the separation of thought from desire and action.

Finally, and to sum up, philosophy may be viewed as a search for self-knowledge, and knowledge of oneself as the highest good. The foregoing pages are in harmony with this doctrine, for individuality involves self-knowledge, and self-knowledge involves both action and reflection. Moreover, both of these are social in character. Socrates, who proclaimed the attainment of self-knowledge to be the highest end of life, exemplified these truths to a high degree. For he sought self-knowledge, not through meditation alone, but chiefly through discourse with his fellow citizens, thus recognizing that self-knowledge is possible only through the knowledge of others. In the end, his own attainment of self-knowledge was proved by his choice to die as he did, an action which expressed his conviction both of the essentially social nature of himself as a citizen, and of the infinite value of a social ideal, in comparison with the existence of a single individual.

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