
APOLLO

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PROJECT APOLLO

APOLLO REENTRY GUIDANCE AND NAVIGATION EQUATIONS AND FLOW LOGIC

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SUMMARY

This report defines the Apollo reentry guidance and navigation (G & N) equations and describes generally the G & N scheme. Program logic, flow charts, and derivations of important guidance equations and range-prediction techniques are included in the figures and appendices. The guidance logic presented applies generally to all Apollo missions; however, minor changes may be necessary to satisfy requirements for a particular mission. The guidance system was designed with performance and flexibility as the primary criteria, and it is anticipated that minor changes will not be difficult to implement.

INTRODUCTION

This report has been prepared to serve as a reference document for programming the Apollo entry guidance equations to be used in the real time computing system in the Mission Control Center and other Apollo entry simulations. Its purpose is to provide a definition of program requirements for IBM.

It was prepared by the Mission Analysis Branch from information contained in references 1 and 2 and from information gained through contacts with Massachusetts Institute of Technology (MIT) personnel. The personal contacts provided interpretation of and revisions to the information in these two references which was not available elsewhere. Thus this document was prepared in order to provide a description of the current version of the entry guidance equations which is not contained in any other report.

The Apollo reentry G & N system as described in references 1 and 2 is an automatic, self-contained inertial system utilizing an onboard digital computer. The objectives are to maintain the Apollo vehicle within a safe flight corridor and to provide guidance to a preselected target location. These mission objectives must be achieved even with imperfect equipment performance and non-standard environmental conditions. The system must operate successfully under these conditions with an inertial measurement unit as its only source of information. These objectives impose requirements on the transearth phase in two ways. The vacuum-perigee altitude deviation must be small enough to insure spacecraft reentry within the acceptable reentry corridor, and the uncertainties in the knowledge of the position and velocity vectors at the start of reentry must be small enough so that their contribution to the final position error at the target is within an acceptable range. Operation within an acceptable flight corridor means that the deceleration during reentry must not exceed some prescribed limit, such as 10 gs, and/or the vehicle should not be allowed to skip back out of the atmosphere at greater than orbital velocity. The guidance does, however, allow a limited skip, which is necessary for range control, but this skip must be performed at sub-circular velocity to prevent excessive flight time.

Since the reentry guidance is capable of completely automatic operation, the flight crew responsibility is to monitor the operation and be prepared at all times to override it in case of malfunction. If a malfunction does occur, the crew becomes a primary control link and follows some simple scheme that ensures a safe-return trajectory.

It is important to realize the major differences between the reentry phase and the other phases of the lunar mission in so far as the guidance problem is concerned. It is the only phase in which there is no subsequent phase which can be used to compensate for errors and uncertainties which result from previous mistakes. It is a phase in which important maneuvers and computations cannot be monitored and checked on the ground because of communication blackout and the rapid response required to control the reentry flight. It is the only phase in which aerodynamic forces, which historically have been much less predicable than rocket forces, are used to control the vehicle. The reentry G & N system must perform its function in such a way that these differences do not penalize the overall mission objectives.

The program described in this document is a general G & N system to be used for all Apollo missions, and does not apply to a specific mission. References 1 and 2 and personal contact with MIT personnel were used as informational sources. Mission-oriented gains, constants, and revisions to satisfy some particular mission requirements will be documented separately. The flow logic, equations, and descriptions presented herein are restricted in the sense that they are interpretations of the MIT reentry guidance equations by the Reentry Studies Section, Mission Analysis Branch, and, therefore, are subject to change on receipt of additional information.

DISCUSSION

The reentry guidance program basically consists of five main control phases, each of which utilize a somewhat different approach to energy management. The five phases are (1) atmospheric entry to guidance initialization (initial roll), (2) constant drag, (3) upcontrol, (4) Kepler, and (5) the suborbital phase (Predict 3). Each of the five main phases and the pertinent subphases are discussed under their respective subtitles and are presented in the form of flow charts which describe the logic and equations that control the reentry trajectory. Figure 1 presents the overall sequence of guidance operations during the reentry flight. Each operation or phase shown in figure 1 is presented in flow diagrams and discussed in the sequence in which the actual control logic flows. The term "drag" will be used throughout this document to mean the total acceleration rather than the acceleration along the velocity vector, which is its explicit and recognized meaning. The symbols for computer variables which are stored in the erasable memory are presented in table I, and the guidance program gains are presented in table II.

Navigation

Every pass through the reentry guidance equations (every 2 seconds) is begun with a section called "navigation." The function of this section is to transform the output from the pulse integrating pendulum accelerometers (PIPAS) into position and velocity vectors which can be used by the guidance program to generate a control command. This section utilizes an average g computation to provide the velocity change due to gravity, and a trapazoidal integration technique to calculate the current position vector. The PIPAS are read every 0.5 second and summed over a 2-second guidance integration step to produce the ΔV input to the navigation equations. The flow logic and equations for this function are presented in figure 2.

Initialization

The initialization routine is entered only once, and this occurs after the G & N entry mode is activated. The G & N entry mode is the primary mode of control for entry into the earth's atmosphere. This mode provides an automatic attitude hold and maneuver capability prior to and throughout the aerodynamic entry. The preentry attitude control phase can receive attitude error information from the G & N system in all three channels. At .05 g, when the entry attitude control phase is activitated by the .05 g switch, the pitch and yaw channels perform rate stabilization only, and the commands are received by the roll channel.

The function of the initialization phase is to compute and preset certain control variables important to program initialization. These functions are as follows: (1) Set the mode selector switch to INITROLL (initial roll phase) or KEP2, depending on the reentry velocity magnitude. The initial roll phase, which is utilized for super-circular velocities, is described in detail later. The KEP2 phase, as distinguished from the Kepler phase, is normally called from the Kepler phase where the only other function of the Kepler phase is to update the mode selector. In the case where KEP2 is entered initially, the update is not necessary; (2) Precompute the vector quantities UTR and UTE, and make an initial estimate of the range-to-go (THETA) based on a previous estimate of the initial target vector (RTINIT). These variables are needed for the first pass through the targeting phase; (3) Preset the variable K2ROLL, which is used by the lateral logic to determine the direction of the initial roll command. To do this requires an initial calculation of the lateral target angle (LATANG), a measure of the lateral range error, and is based on the previous estimate of the initial target vector (RTINIT). K2ROLL is then set equal to minus LATANG, and an initial roll command of 15° toward the target results; (4) Determine if the initial roll angle, which depends upon initial velocity and flight-path angle at reentry, should result in lift vector up or down; (5) Call the routine which calculates the desired attitude of the vehicle independent of the main steering commands. This routine is called the preentry attitude control phase. This routine calculates the commanded gimbal angles by computing the desired orientation of the navigation base with respect to the relative velocity vector. The flow logic and equations for this phase are presented in figure 3.

Targeting

Targeting, as the name implies, is the process of determining the target coordinates on a rotating earth, and the location and motion of the vehicle with respect to these coordinates. This information is mandatory to the operation of the closed-loop guidance system and is calculated every pass through the guidance logic. At a preselected value of velocity (VMIN) the targeting will switch to relative coordinates. The parameters which are calculated in this phase are (1) the predicted target vector (URT), which accounts for a predicted earth rotation based on an estimated flight time, (2) the range-to-go (THETA), which is simply the arc cosine of the dot product of the position vector ($\vec{R}$) of the vehicle and the predicted target vector ($\vec{RT}$), and (3) the lateral angle (LATANG), which represents the angle between the target vector and a radius vector formed by the intersection of the osculating plane and a plane which contains the target vector ($\vec{RT}$) and is perpendicular to the osculating plane. This angle is used in the lateral logic to control the direction of the roll command. The flow logic and equations are presented in figure 4.

Initial Roll Phase

Atmospheric entry is assumed to occur at .05 g, at which time the entry attitude control mode is activated. This is a mode which will accept commands only in the roll channel and utilizes the pitch and yaw channels for damping undesirable rates. The function of the initial roll phase is simply to test a roll-up switch in order to initiate a roll to a lift-vector-up attitude at a preselected value of load factor, and to initiate closed-loop guidance control when the altitude rate is equal to a preset value (VRCNTR). This flow logic is presented in figure 5.

Huntest

Maintenance of the vehicle within the safe flight corridor is the prime constraint of the energy management system during super-orbital flight; however, range control is considered mandatory because the major part of the vehicle's ranging capabilities exist during this velocity regime. In this regime, range control is the primary control requirement, within acceptable limits. The acceptable limits are determined by the maximum load factor and the exit velocity when skip trajectories are required. During super-orbital flight the energy management system satisfies the range requirements by the proper selection of the exit velocity (VL) and the corresponding exit flight-path angle (GAMMAL). To calculate the exit velocity, the assumption is made that drag varies paribolically with velocity. A solution requires only two independent variables if the exit drag (Q7) is constant. The two independent variables are the initial up-control velocity (V1) and the initial up-control drag (A0). The derivations of the necessary equations are presented in appendix A. The Huntest phase, in which the exit velocity and flight-path angle are calculated, is initiated at the same time that closed-loop guidance is initiated, i.e., when the altitude rate has been reduced to a preselected value (VCNTR). If the vehicle has a negative flight-path angle when Huntest is initiated, the initial values of up-control velocity (V1) and drag (A0) are selected such that they occur at the point where the flight-path angle will become zero (pull-out). If the flight-path angle is positive when Huntest is initiated, V1 and A0 would be assigned the actual values of the velocity and drag of the vehicle. The exit velocity (VL) is then computed with equations which neglect the effect of gravity and centrifugal force. If the exit velocity limitations are violated, such that VL is larger than circular velocity (VSAT), the constant drag phase is used to provide a control command based on the selected value of A0, until a new VL can be calculated. This is done by perturbing the initial value of up-control velocity (V1) and up-control drag (A0) and then computing a new VL. This procedure is repeated until an acceptable value of VL has been determined. A corresponding value of the exit flight-path angle (GAMMAL) is then computed by an equation which includes the effect of the acceleration due to gravity and centrifugal force to determine if the predicted exit conditions can physically be obtained (see appendix B for derivation of equations). If not, the apogee is assumed to occur within the sensible atmosphere, and new values of VL and Q7 which are calculated represent the velocity and drag at apogee. If VL is below a preselected minimum (VLMIN), the guidance will phase directly into the second entry phase (Predict 3). When all velocity and flight-path angle requirements have been satisfied, the guidance proceeds to the range-prediction phase to satisfy the range requirements. The flow logic and equations are presented in figure 6.

Range Prediction

Because of the different energy management schemes used, the predicted range is calculated for each phase and summed for the total predicted range-to-go. The basic concept used for calculating predicted range for the pre-up phase and the up-phase is simply the velocity times the delta time. The derivations are presented in appendix D. The Kepler range prediction is made by combining two equations which give the range to apogee from any point in orbit and multiplying by two for the total range. This derivation is presented in appendix D, also. Since the sub-orbital phase guides to a stored reference trajectory which has a fixed range, the predicted range is based upon linear perturbations about the stored reference trajectory. A correction must be made to the sub-orbital phase range prediction to account for a possible difference in initial flight-path angle between the reference trajectory and the actual trajectory. This correction is calculated by multiplying the delta flight-path angle by a sensitivity coefficient of range with respect to flight-path angle. The total predicted range is compared with the actual range-to-go, and, if the predicted range is less than the actual by a value larger than the accepted tolerance (25 n. mi.), the range prediction phase will perturb the value of V1 and return to the Huntest phase to calculate a new VL and repeat the above sequence of calculations. If the predicted range is greater than the actual by a value larger than the tolerance, the guidance will command a constant drag trajectory, based on AO, until the predicted range becomes less than the actual, or the range requirement is satisfied. The guidance will then proceed to the up-control phase and guide to the value of exit velocity it has chosen. The flow logic and equations are presented in figure 7.

Constant Drag

The function of the constant drag phase is to provide guidance for the vehicle between the initialization of the closed-loop guidance and the start of range control guidance. When the guidance control logic is phasing in and out of the Huntest or range prediction phase to satisfy range and exit velocity requirements, the constant drag phase will guide to a drag value which is set in Huntest. This value of drag is the initial up-control drag (AO), which is redesignated DO for this phase, and restricted to remain less than a load factor of 10 and greater than a minimum designated by the input constant C19. The control procedure is to command an L/D which is based upon the difference between a reference drag and the altitude rate required to maintain a constant load factor and the actual drag and altitude rate. This type of control will cause the actual trajectory to oscillate about the reference drag (DO). The rate of convergence is a function of the gains involved in the equation for L/D. The flow logic and equations are presented in figure 8.

Up Control

The function of the up-control phase is to provide guidance control for that part of the reentry maneuver from the termination of Huntest to the initialization of the suborbital or Kepler phases. Range control during this phase is a prime requisite of the overall energy management system because the major part of the vehicle's ranging capability exists within the velocity regime encompassed by this phase. The dependent variables are the reference velocity (VREF) and the reference flight-path angle (GAMMALREF). The derivation of the guidance equations (which are based upon this assumption) are presented in appendices A and B. The L/D commanded is computed from the difference between the vehicle velocity and flight-path angle and the reference velocity and flight-path angle. The acceleration due to gravity and centrifugal force is considered at velocities less than circular orbital velocity. If the vehicle velocity is greater than the initial up-control velocity (V1) when the up-control phase is initiated, a pre-up control command will be based upon a maximum lift trajectory to pull-out, which was the basic assumption when V1 and AO were initially computed. The derivations of the equations to calculate the reference values of altitude rate and drag are presented in appendix C. If, at any time, the vehicle drag becomes greater than AO, lift-vector-up will be commanded until the drag becomes less than AO again. The flow logic and equations are presented in figure 9.

Kepler

The function of the Kepler phase is to provide a guidance logic for that part of the trajectory which exceeds the sensible atmosphere in order to satisfy ranging requirements. The guidance logic commands the preentry attitude control phase under these conditions, and the function of the Kepler phase logic is to monitor the spacecraft resultant drag or load factor to determine when to restart lift vector control. The attitude hold mode allows attitude error signals to be received in all three channels--pitch, yaw, and roll--such that a given attitude can be maintained throughout the region in which there are no aerodynamic forces. The flow logic and equations are presented in figure 10.

Predict 3

The Predict 3, or suborbital phase, controls the vehicle from the termination of the Kepler phase, or the first apogee point if the Kepler phase is not used, to drogue chute deployment. Unlike the other phases, Predict 3 utilizes linear perturbation about a stored reference trajectory to calculate L/D commands. The reference trajectory is based on a constant L/D and, therefore, a constant bank angle; thus, the suborbital guidance system logic does not guide the vehicle back to a reference trajectory, but on a path which will converge to the same end point. The reference trajectory is presented in table 3 in the form used by the guidance equations. The Predict 3 flow diagram is shown in figure 11.

The other important tasks which are performed by this phase are to stop guidance control below a preset limiting velocity and to command a lift-vector-down if the vehicle passes the target prior to drogue deployment. A g-limiter logic is entered after each pass through Predict 3. The function of the g-limiter is to monitor the current load factor and to assume control to prevent excessive loads if it exceeds one half of the maximum allowable load factor. The flow logic and equations are presented in figure 12.

Lateral Control

The vehicle is controlled by rolling the lift vector about the velocity vector such that the vertical component controls the down-range landing point by varying from lift-vector-up to lift-vector-down. This process of rolling the vehicle results not only in down-range control, but also cross-range control, because of the coupling between the vertical and side forces. It is the function of the lateral control phase to monitor the predicted lateral landing error (LATANG) and command a roll direction change when this parameter exceeds a computed deadband (Y), which is a function of the velocity squared and, therefore, becomes smaller as velocity decreases. Since this is the last phase in the reentry guidance and navigation system flow logic, the roll command is computed and control is transferred to the stabilization and control system to implement the command roll angle. The flow logic and equations are presented in figure 13.

REFERENCES

1. Dahlen, J.; Kosmala, A.; Lickly, D.; Morth, R.; Shillingford, J.; and Sokkappa, B.: Guidance and Navigation System Operations Plan Apollo Mission 202. R-477, Rev. 2, MIT Instrumentation Laboratory, October 1965 (Confidential).

2. Lawhorn, Thomas W.; Lee, Eva A.; Vaughan, Robert D.; and Williams, John E., Jr.: Flow Diagrams of Corona 216, Apollo Flight 202A Software. MSC IN 65-EG-46, Preliminary Release, November 3, 1965.

3. Thomson, William Tyrell: Introduction to Space Dynamics, John Wiley and Sons, Inc., New York, 1961.

APPENDIX A

DERIVATION OF UP CONTROL REFERENCE VELOCITY

The equations of motion for the reentry vehicle are:

$$\frac{dV}{dt} = -\frac{1}{2} \rho V^2 \frac{SC_D}{M} - G \sin\gamma \quad (1)$$

$$\frac{dV}{dt} = \left(\frac{V^2}{R} - G\right) \cos\gamma + \frac{1}{2} \rho V^2 \frac{SC_L}{M} \quad (2)$$

$$\frac{dH}{dt} = V \sin\gamma = \dot{R} \quad (3)$$

If $\sin\gamma = 0$ and $\cos\gamma = 1$, the above equations may be written as:

$$\frac{dV}{dt} = -D \quad (4)$$

$$V \frac{d\gamma}{dt} = (\bar{V}^2 - 1) G + L = \sin\gamma \ dV + V \cos\gamma \ d\gamma \quad (5)$$

and, therefore,

$$\frac{d\dot{R}}{dt} = (\bar{V}^2 - 1) G + L \quad (6)$$

If centrifugal and gravity accelerations are neglected, equation (6) becomes

$$(\bar{V}^2 - 1) G = 0 \quad (7)$$

and, by eliminating time between Equations (4) and (6),

$$\frac{d\dot{R}}{dV} = -\frac{L}{D} \quad (8)$$

which can be integrated starting at

$V = V1$, and $\dot{R} = 0$ for constant L/D

$$\dot{R} = L/D \ (V1 - V) \quad (9)$$

Now we can solve for altitude as a function of velocity by combining equations (3), (4), and (9) to eliminate time

$$\frac{dH}{dV} = - \frac{L}{D} \frac{(V1 - V)}{D} \qquad (10)$$

Now relate drag to velocity

$$D = \frac{1}{2} \rho V^2 \frac{SC_D}{M} \qquad (11)$$

and density to altitude by assuming exponential variation of density with altitude

$$\rho = \rho_1 e^{-\beta (H - H1)} \qquad (12)$$

Substitute Equations (11) and (12) into Equation (10) to get

$$\frac{dH}{dV} = - \frac{L/D\,(V1 - V)}{\frac{1}{2} \rho_1 e^{-\beta (H - H1)} \frac{SC_D V^2}{M}} \qquad (13)$$

This equation integrates directly to

$$\left(\frac{SC_D}{M}\right)\left(\frac{1}{L/D}\right) \frac{1}{\beta} \left(- \frac{1}{2} \rho_1 e^{-\beta (H - H1)} + \frac{1}{2} \rho_1\right) = - \ln\left(\frac{V1}{V}\right) + \frac{V1}{V} - 1 \qquad (14)$$

The right hand side can be simplified by

$$- \ln\left(\frac{V1}{V}\right) + \frac{V1}{V} - 1 = - \ln (1 + \Lambda) + \Lambda \qquad (15)$$

where

$$\Lambda = \frac{V1 - V}{V}$$

The log function can be expanded in a series

$$\ln (1 + \Lambda) = \Lambda + \frac{\Lambda^2}{2} + \ldots$$

so that

$$- \ln\left(\frac{V1}{V}\right) + \frac{V1}{V} - 1 = -\Lambda + \frac{\Lambda^2}{2} + \ldots + \Lambda = \frac{\Lambda^2}{2}$$
$$= \frac{1}{2} \left(\frac{V1 - V}{V}\right)^2$$

Put this result back into Equation (14)

$$\frac{1}{(L/D)\beta} \quad \frac{SC_D}{M}\left(-\frac{1}{2}\ \rho + \frac{1}{2}\,\rho_1\right) = \frac{1}{2}\left(\frac{V1 - V}{V}\right)^2$$

Use Equation (11) to eliminate ρ & ρ_1

$$\frac{1}{(L/D)\beta}\left(- D + D1\,\frac{V^2}{V1^2}\right) = \frac{1}{2}\,(V1 - V)^2 \qquad (16)$$

To solve for V in terms of D regroup Equation (16) into a quadratic in V

$$V^2\left(\frac{2D1}{\beta V1^2} - L/D\right) + 2\,(L/D)\ V1\ V - \frac{2D}{\beta}\ -\left(\frac{L}{D}\right) V1^2 = 0$$

Rewrite this equation

$$X^2\ (L/D)\ (\alpha - 1) + 2\ (L/D)\ X - L/D\left(\alpha\frac{D}{D1} + 1\right) = 0$$

where $X = \frac{V}{V1}$

and

$$\alpha = \frac{2D1}{\beta V1^2 (L/D)}$$

Use the quadratic formula

$$X = -\frac{2(L/D)}{2(L/D)(\alpha - 1)} \pm \sqrt{\frac{4(L/D)^2}{4(\frac{L}{D})^2(\alpha - 1)^2} + \frac{L/D(\alpha\,\frac{D}{D1} + 1)}{L/D\ (\alpha - 1)}}$$

Noting that the negative sign on the square root is appropriate, this can be simplified to

$$X = \frac{1}{1 - \alpha}\left[1 - \sqrt{(\alpha - 1)\alpha\,\frac{D}{D1} + \alpha}\right] \qquad (17)$$

Finally summarize result by rewriting in the form used in the guidance

$$VREF = FACT1\ (1\ \sqrt{FACT2\ D + ALP}) \qquad (18)$$

where

$$ALP = \frac{2D1}{\beta\ V1^2 (L/D)}$$

and if

$$AO = D1$$
$$\beta = 1/HS$$
$$ALP = \frac{2HS\ AO}{(L/D)V1^2} \qquad (19)$$

and

$$FACT1 = \frac{V1}{1 - ALP} \qquad (20)$$

$$FACT2 = \frac{ALP\ (ALP - 1)}{AO} \qquad (21)$$

APPENDIX B

DERIVATION OF EQUATIONS WITH GRAVITY AND CENTRIFUGAL FORCE EFFECT

The calculation for GAMMAL which allows for the previously neglected gravity and centrifugal acceleration is as follows:

Start again with basic equations, this time including gravity and centrifugal acceleration.

$$\frac{dV}{dt} = - k1 \; \rho_o \; e^{-\beta H} V^2 \qquad (1)$$

$$\frac{d\dot{R}}{dt} = k2 \; \rho_o \; e^{-\beta H} V^2 + G \; (\bar{V}^2 - 1) \qquad (2)$$

$$\frac{dH}{dt} = \dot{R} \qquad (3)$$

where for Equation (2)

$$\dot{R} = V \sin \gamma$$

$$\frac{d\dot{R}}{dt} = V \cos \gamma \frac{d\gamma}{dt} + \sin \gamma \frac{dV}{dt}$$

and, if $\sin \gamma = 0$ and $\cos \gamma = 1$,

$$\frac{d\dot{R}}{dt} = V \frac{d\gamma}{dt}$$

Divide (2) by (1)

$$\frac{d\dot{R}}{dV} = - \frac{k2}{k1} + \frac{G}{k1 \; e^{-\beta H}} \left[\frac{1}{V^2} - \frac{1}{V_S^2}\right] \qquad (4)$$

Note that $k2/k1 = L/D$

Equations (3) and (4) are a non-linear set and, thus, approximations are in order. The approximation is to assume a straight line variation of $e^{\beta H}$ with velocity.

$$e^{\beta H} = 1 + a \; (V_S - V) \qquad (5)$$

The constant "a" is yet to be determined. This approximation yields for Equation (4)

$$\frac{d\dot{R}}{dV} = -\frac{L}{D} + \frac{G}{k1}\left[(1 + a(VS - V))\left(\frac{1}{V^2} - \frac{1}{VS^2}\right)\right] \tag{6}$$

This is readily integrated starting at VS to V

$$\dot{R} = \dot{R}_o + \frac{L}{D}\ (VS - V) + \frac{G}{k1}\left\{(1 + a\ VS)\left[-\frac{1}{V} + \frac{1}{VS}\right] - a \ln\frac{V}{VS} + \frac{1 + a\ VS}{VS^2}(VS - V) + \frac{aV^2}{2VS^2} - \frac{a}{2}\right\} \tag{7}$$

As before, we can expand the log function thus

$$\ln\frac{V}{VS} \cong \frac{V - VS}{VS} - \frac{1}{2}\left(\frac{V - VS}{VS}\right)^2$$

This substitution yields, after some algebra,

$$\dot{R} = \left[\dot{R}_o + L/D\ (VS - V)\right] + \frac{G}{DoV}\left[-(VS - V)^2 - a\ (VS - V)^3\right] \tag{8}$$

Note that

$$\dot{R}_o + \frac{L}{D}(VS - V) = LEWD\ (V1 - V) = RDOT1 \tag{9}$$

and

$$GAMMAL1 = \frac{RDOT1}{DVL}$$

Now return to determine the constant a. The drag is assumed to vary parabolically with velocity, no gravity correction. This yields an exit velocity (VL) for the exit drag (Q7). A straight line is then drawn between the corresponding densities at VS and VL. The desired slope is then

$$a = \frac{(DHOOK - Q7)}{Q7} / (VS - V1) \tag{10}$$

where DHOOK is obtained by solving Equation (18A) for D.

When the solution of Equation (8) becomes negative or zero, this signifies a peak altitude for the exit portion, and the exit acceleration (Q7) can not be reached because the acceleration due to gravity and centrifugal force is too great. This effect is implemented as follows.

If $\dot{R}$ is negative on the particular computation cycle, the derivative of Equation (8) with respect to velocity ($d\dot{R}/dV$) is evaluated at VL by

$$\frac{d\dot{R}}{dV} \approx (-3a\ \delta V - 2\ \delta V)\ \frac{G}{DoVL} + LEWD \qquad (11)$$

where

$$\delta V = (VS - VL)$$

V2, the velocity at which $\dot{R}$ becomes zero, is calculated by using this derivative; thus,

$$V2 = VL + \frac{\dot{R}L}{d\dot{R}/dV} \qquad (12)$$

This V2 is set equal to the exit velocity (VL) and a new exit acceleration is calculated by the usual parabolic relation

$$Q7 = \left[\left(1 - \frac{V_L}{FACT1}\right)^2 - ALP\right] /FACT2 \qquad (13)$$

This Q7 is used for reference calculations on future computational cycles. Finally the exit altitude rate $\dot{R}$ is set equal to zero.

APPENDIX C

DERIVATIONS OF EQUATIONS FOR PRE-UP PHASE

Velocity and drag conditions at the time of entry to the pre-up phase are projected to V1 and A0 (velocity and drag at the start of up-control) at zero vertical velocity by assuming flight at maximum L/D. The same reasoning as for the up-control phase is used, that is, RDOT varies linearly with velocity; so from Equation (9A):

$$V1 = V + RDTR/(L/D)_{max}$$

or

$$RDTR = -(V - V1)\ (L/D)_{max} \qquad (1)$$

where

$$RDTR = \dot{R}$$

The increase in acceleration is calculated by

$$A0 = DR + \delta D \qquad (2)$$

where

DR = Reference Drag

Assume constant velocity to calculate δD where

$$D = \frac{1}{2} \rho V^2 \frac{C_D S}{M}$$

and, if

$$\rho = \rho_0 e^{-H/HS} \qquad (3)$$

then

$$\delta D = -\frac{DR \delta H}{H_s}$$

Also,

$$\delta H = \dot{R}_{AVE}\ \delta T \qquad (4)$$

$$\dot{R}_{AVE} = \dot{R}/2 \qquad (5)$$

and

$$\delta T = \dot{R}/(L/D)_{max}D \qquad (6)$$

Summarizing

$$AO = DR + \frac{RDTR^2}{2Hs(L/D)_{max}} \qquad (7)$$

APPENDIX D

DERIVATIONS OF RANGE PREDICTION EQUATIONS

Pre-Up Phase

The range for this phase is calculated by

$$\text{Range} = \frac{V\delta t}{R_E} \tag{1}$$

where

R_E = Earth Radius

This time by assuming velocity constant Equation (6A) yields:

$$\delta t = \frac{\dot{R}}{(L/D)_{max} AD} \tag{2}$$

and

$$\text{Range} = \dot{R}V/(L/D)_{max} AO\ R_E \tag{3}$$

Up-Control Phase

Again, assume that

$$\text{Range} = \frac{VI\delta t}{R_E} \tag{4}$$

where

$$\delta t = \frac{\delta H}{\dot{R}L} \tag{5}$$

Now relate drag to velocity and density to altitude by

$$D = \frac{1}{2} \rho V^2 \frac{SCD}{M} \tag{6}$$

and

$$\rho = \rho_0\, e^{-H/HS} \tag{7}$$

By differentiating Equation (6)

$$dD = -\frac{1}{2}\ \rho V^2\ \frac{C_D S}{M}\ \frac{dH}{HS} + \frac{1}{2}\ \rho V\ \frac{C_D S}{M}\ dV$$

and solving for dH

$$dH = \left[-\frac{dD}{D} + \frac{2DV}{V}\right]\ HS \tag{8}$$

By substituting into Equation (5)

$$\delta t = \frac{HS}{\dot{R}_L}\left[-\frac{dD}{D} + \frac{2dV}{V}\right] \tag{9}$$

and Equation (4)

$$\text{Range} = \frac{HS}{R_E}\ \frac{V_L}{\dot{R}_L}\left[-\frac{dD}{D} + \frac{2dV}{V}\right]$$

Finally by integrating across the up-control phase

$$\text{Range} = \frac{1}{R}\ \frac{HS}{\text{GAM[illegible]1}}\quad \ln\left[\frac{AO\ V_L^2}{Q7\ V1^2}\right] \tag{10}$$

<u>Kepler Phase</u>

From Reference 3

$$\tan\left(\frac{\theta}{2}\right) = -\frac{(RV^2/k)\sin\gamma\ \cos\gamma}{(RV^2/k)\cos^2\gamma - 1} \tag{11}$$

and

$$\cos\left(\frac{\theta}{2}\right) = \frac{R - \dfrac{(RV\ \cos\gamma)^2}{\mu}}{Re} \tag{12}$$

where

$$\frac{R}{k} = \frac{1}{gR}$$

$$\frac{V^2}{Rg} = \bar{V}^2$$

$$e = \sqrt{1 - 2\ \bar{V}^2\cos^2\gamma + \bar{V}^4\ \cos^2\gamma}$$

$\mu = R^2 g$

therefore,

$$\tan\left(\frac{\theta}{2}\right) = \frac{\bar{V}^2 \sin\gamma \cos\gamma}{\bar{V}^2 \cos^2\gamma - 1} \tag{13}$$

and

$$\cos\left(\frac{\theta}{2}\right) = \frac{1 - \bar{V}^2 \cos^2\gamma}{e} \tag{14}$$

Now

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta/2)}{\cos(\theta/2)} \tag{15}$$

so, by substitution,

$$\sin\left(\frac{\theta}{2}\right) = \frac{\bar{V}^2 \sin\gamma \cos\gamma}{e} \tag{16}$$

and, therefore,

$$\text{Range} = 2 \sin^{-1}\left[\frac{\bar{V}^2 \sin\gamma \cos\gamma}{e}\right] \tag{17}$$

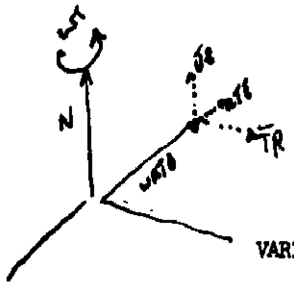

TABLE I

VARIABLES FOR REENTRY CONTROL

$U\overline{R}TO$	INITIAL TARGET VECTOR
$\overline{U}Z$	UNIT VECTOR NORTH
$\overline{V}$	VELOCITY VECTOR
$\overline{R}$	POSITION VECTOR
$\overline{R}TE$	VECTOR EAST AT INITIAL TARGET
$\overline{U}TR$	NORMAL TO $\overline{R}TE$ AND $\overline{U}Z$
$\overline{W}E$	EARTH RATE VECTOR
$U\overline{R}T$	TARGET VECTOR
$\overline{U}NI$	UNIT NORMAL TO TRAJECTORY PLANE
$\overline{D}ELV$	INTEGRATED ACCELERATION VECTOR
$\overline{G}$	GRAVITY VECTOR
AO	INITIAL DRAG FOR UPCONTRL
AHOOK	TERM IN GAMMAL COMPUTATION
ALP	CONST FOR UPCONTRL
ASKEP	KEPLER RANGE
ASP1	FINAL PHASE RANGE
ASPUP	UPRANGE
ASP3	GAMMA CORRECTION
ASPDWN	RANGE DOWN TO PULL-UP
ASP	PREDICTED RANGE = ASKEP+ASP1+ASPUP+ASP3+ASPDWN
COSG	COSINE (GAMMAL)
D	TOTAL ACCELERATION
DO	CONTROLLED CONST DRAG
DHOOK	TERM IN GAMMAL COMPUTATION
DIFF	THETNG-ASP (RANGE DIFFERENCE)
DIFFOLD	PREVIOUS VALUE OF DIFF
DR	REFERENCE DRAG FOR DOWNCONTROL
DREF	REFERENCE DRAG
DVL	VS1 -VL

TABLE I (cont'd)

E	ECCENTRICITY	
F1	DRANGE/D DRAG	(FINAL PHASE)
F2	DRANGE/DRDOT	(FINAL PHASE)
F3	DRANGE/D(L/D)	(FINAL PHASE)
FACT1	CONST FOR UPCONTRL	
FACT2	CONST FOR UPCONTRL	
FACTOR	USED IN UPCONTRL	
GAMMAL	FLIGHT PATH ANGLE AT VL	
GAMMAL1	SIMPLE FORM OF GAMMAL	
KA	ACCELERATION LEVEL TO ROLL LIFT UP	
K1ROLL	INDICATOR FOR ROLL SWITCH	
K2ROLL	INDICATOR FOR ROLL SWITCH	
LATANG	LATERAL RANGE	
LEQ	EXCESS C.F. OVER GRAV = (VSQ-1) GS	
L/D	DESIRED LIFT TO DRAG RATIO (VERTICAL PLANE)	
PREDANGL	PREDICTED RANGE	(FINAL PHASE)
Q7	MINIMUM DRAG FOR UPCONTROL	
RDOT	ALTITUDE RATE	
RDOTREF	REFERENCE RDOT FOR UPCONTRL	
RDTR	REFERENCE RDOT FOR DOWNCONTRL	
ROLLC	ROLL COMMAND	
RTOGO	RANGE TO GO	(FINAL PHASE)
SL	SIN OF LATITUDE	
T	TIME	
THETA	DESIRED RANGE (RADIANS)	
THETNM	DESIRED RANGE (NM)	
V	VELOCITY MAGNITUDE	
V1	INITIAL VELOCITY FOR UPCONTRL	
V1OLD	PREVIOUS VALUE OF V1	

TABLE I (cont'd)

VCORR	VELOCITY CORRECTION FOR UPCONTRL
VL	EXIT VELOCITY FOR UPCONTRL
VS1	VSAT OR V1, WHICHEVER IS SMALLER
VBARS	$VL^2/VSAT^2$
VSQ	NORMALIZED VELOCITY SQUARED = $V^2/VSAT^2$
WT	EARTH RATE X TIME
X	INTERMEDIATE VARIABLE USED IN G LIMITER
Y	LATERAL MISS LIMIT

SWITCHES		INITIAL STATE
RELVELSW	RELATIVE VELOCITY SWITCH	BRANCH (1)
EGSW	FINAL PHASE SWITCH	NON-BRANCH (0)
HUNTIND	INITIAL PASS THRU HUNTEST	NON-BRANCH (0)
HIND	INDICATES INTERATION IN HUNTEST	NON-BRANCH (0)
LATSW	NO LATERAL CONTROL WHEN ON	NON-BRANCH (0)
GONEPAST	INDICATES OVERSHOOT OF TARGET	NON-BRANCH (0)

TABLE 2

GUIDANCE GAINS AND CONSTANTS

SYMBOL	
C1	Factor in ALP computation
C10	Initial roll angle
C12	Factor in ASPUP computation
C16	Constant drag gain (on drag)
C17	Constant drag gain (on RDOT)
C18	Lead velocity for upcontrol start
C19	Minimum constant drag
C20	Minimum D for lift up
CHOOK	Factor in AHOOK computation
CH1	Factor in GAMMAL computation
GMAX	G-limit
KAFIX	Minimum drag for lift-up if down
KB3	Up control gain, optimized
KB4	Up control gain, optimized
KC1	Factor in V1 computation
KC2	Factor in A0 computation
KC3	Factor in ASPDWD computation
KLAT	Lateral switch gain
K13P	Factor in L/D computation for final phase
KOMIN	Increment to Q7 to end Kepler
KTETA	time of flight calculation gain
LAD	Max L/D
LATBIAS	Lateral switch bias term
L/DCMINR	LAD COS (15°)
LEWD	Up control L/D
LOD	Final phase L/D
25NM	Acceptable tolerance to stop range iteration
Q2	Final phase range (-23500 Q3)
Q3	Final phase dR/dV
Q4	Final phase initial velocity
Q5	Final phase dR/d RDOT

TABLE 2 (cont'd)

Q6	Final phase initial RDOT
Q7F	Minimum drag for up control
Q9	KTETA
VCORLIM	Limit value of VCORR
VRCONTRL	Minimum RDOT to close loop
VMIN	Velocity to switch to relative velocity
VLMIN	Minimum VL
VQUIT	Velocity to stop steering
DOMAX	Max drag for constant drag phase
DMIN	drag value for initialise Kepler
LYD	L/D for computing total accel.
HDOTIND	RDOT correction switch

TABLE 3

FINAL PHASE REFERENCE TRAJECTORY

N	VREF	RDOTREF	AREF	DR/DRDOT F2 NM/FPS	DR/DA F1 NM/FPSS	RTOGO	DR/DL/D F3 NM
16	FPS	FPS	FPSS			NM	
15	0	-331	34.1	0	-.02695	0	1
14	337	-331	34.1	0	-.02695	0	1
13	1080	-693	42.6	.002591	-.03629	2.7	6.44 x 2
12	2103	-719	60.	.003582	-.05551	8.9	10.91 x 2
11	3922	-694	81.5	.007039	-.09034	22.1	21.64 x 2
10	6295	-609	93.9	.01446	-.1410	46.3	48.35 x 2
9	8531	-493	98.5	.02479	-.1978	75.4	93.72 x 2
8	10101	-416	102.3	.03391	-.2372	99.9	141.1 x 2
7	14014	-352	118.7	.06139	-.3305	170.9	329.4
6	15951	-416	125.2	.07683	-.3605	210.3	465.5
5	18357	-566	120.4	.09982	-.4956	266.8	682.7
4	20829	-781	95.4	.1335	-.6483	344.3	980.5
3	23090	-927	28.1	.2175	-2.021	504.8	1385
2	23500	-820	6.4	.3046	-7.569	643.0	1508
1	35000	-820	6.4	.3046	-7.569	643.0	1508

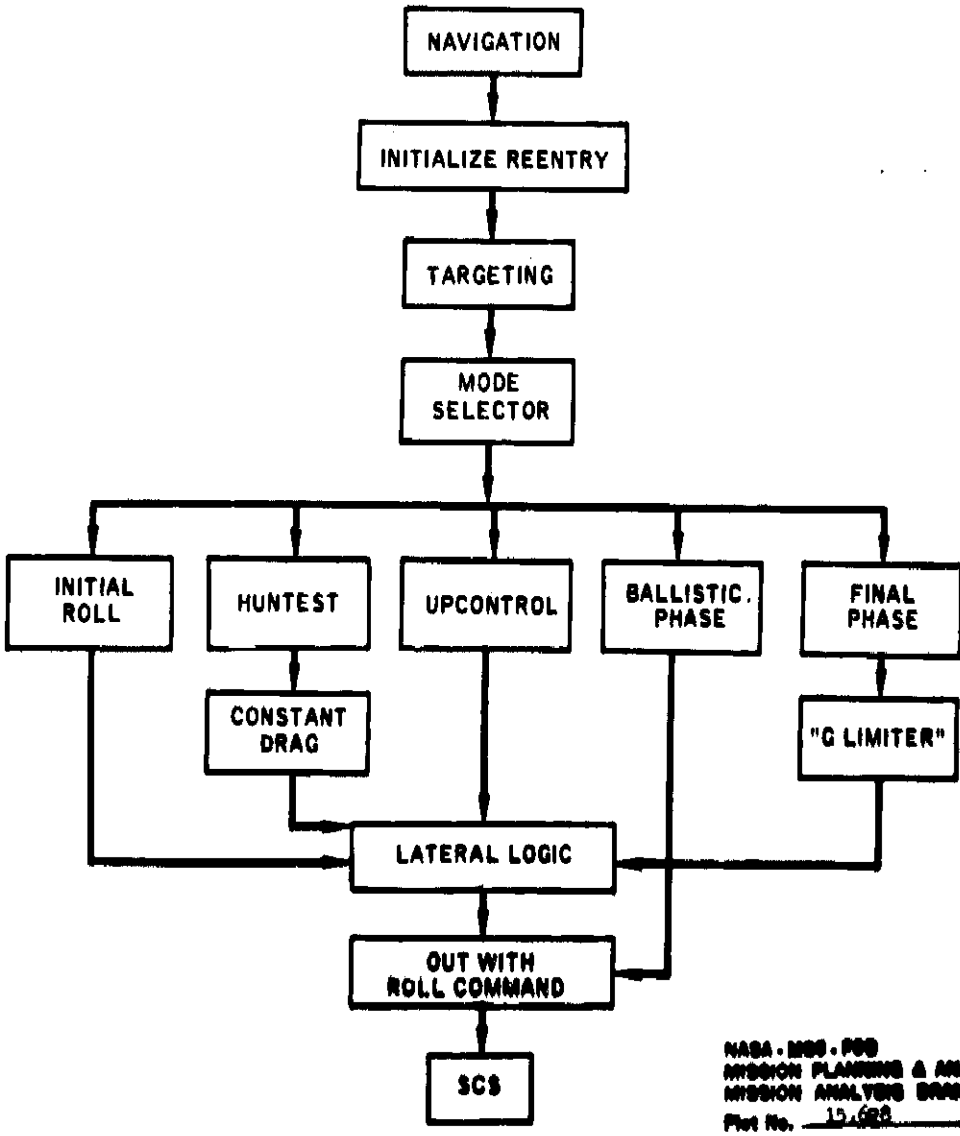

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FIGURE 1.- REENTRY STEERING

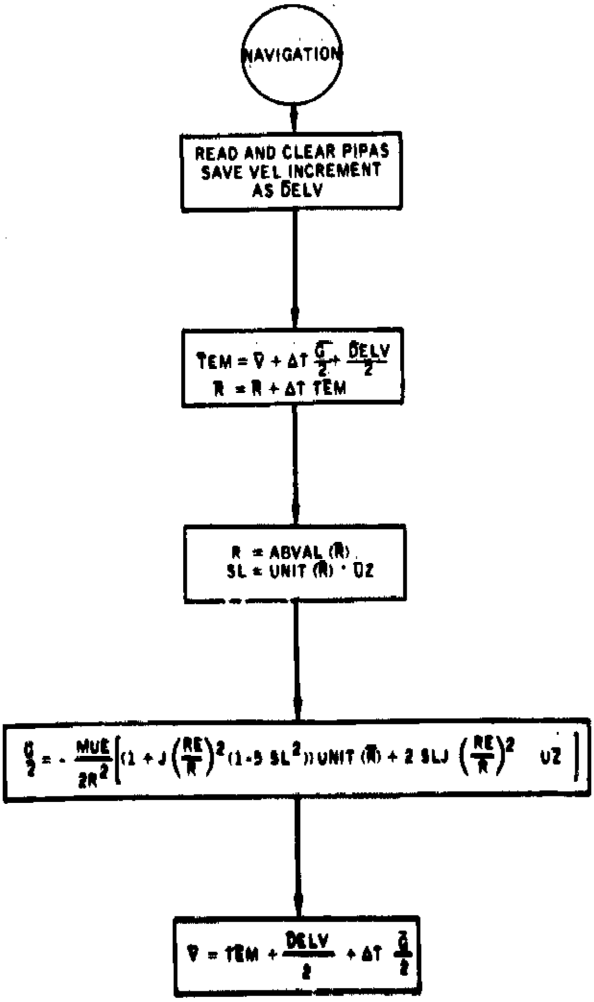

FIGURE 2.- NAVIGATION ΔV COMPUTATION.

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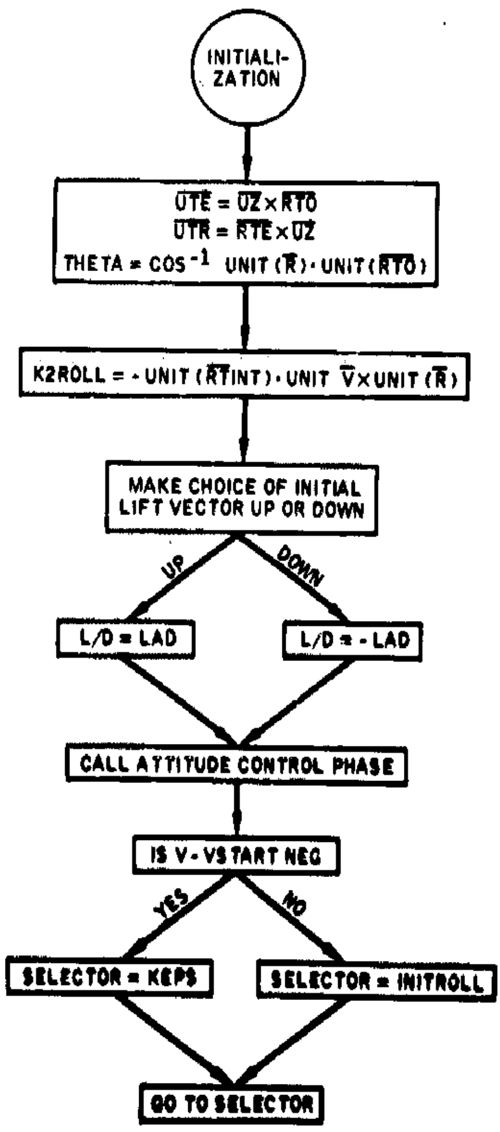

FIGURE 3. - INITIALIZATION

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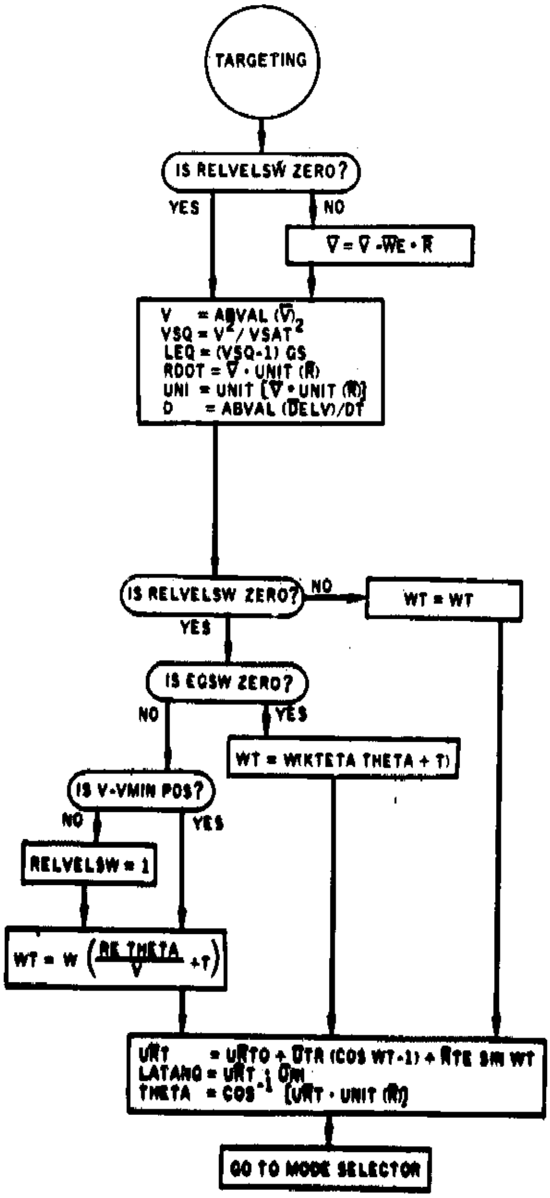

FIGURE 4.- TARGETING

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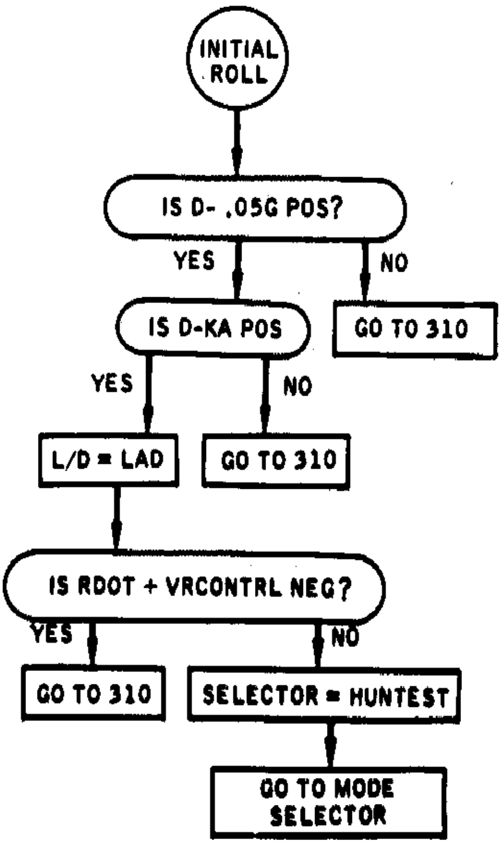

FIGURE 5.- INITIAL ROLL

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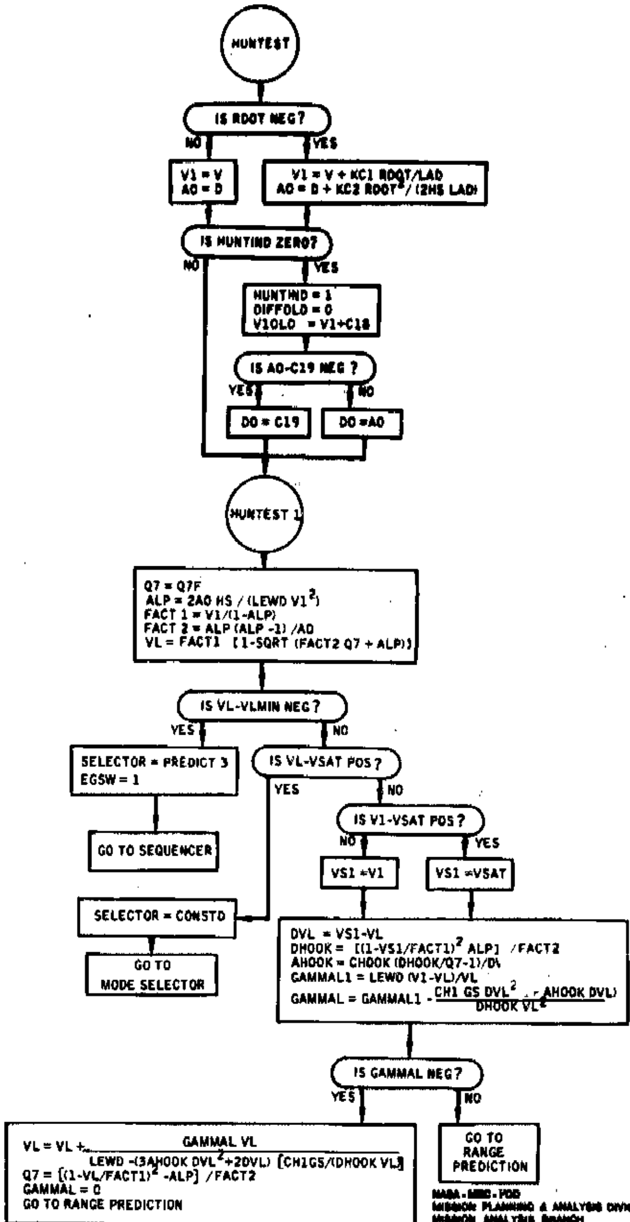

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FIGURE 6.- HUNTEST

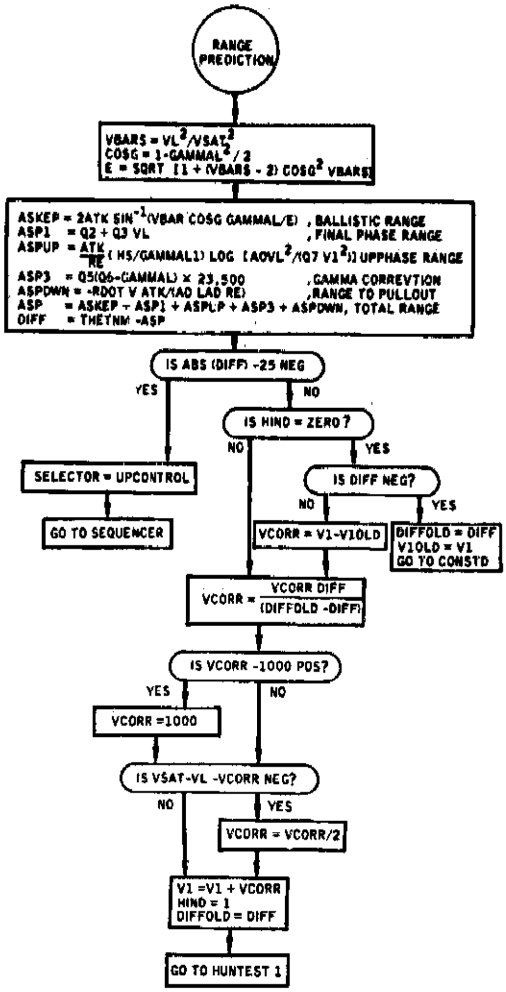

FIGURE 7.- RANGE PREDICTION

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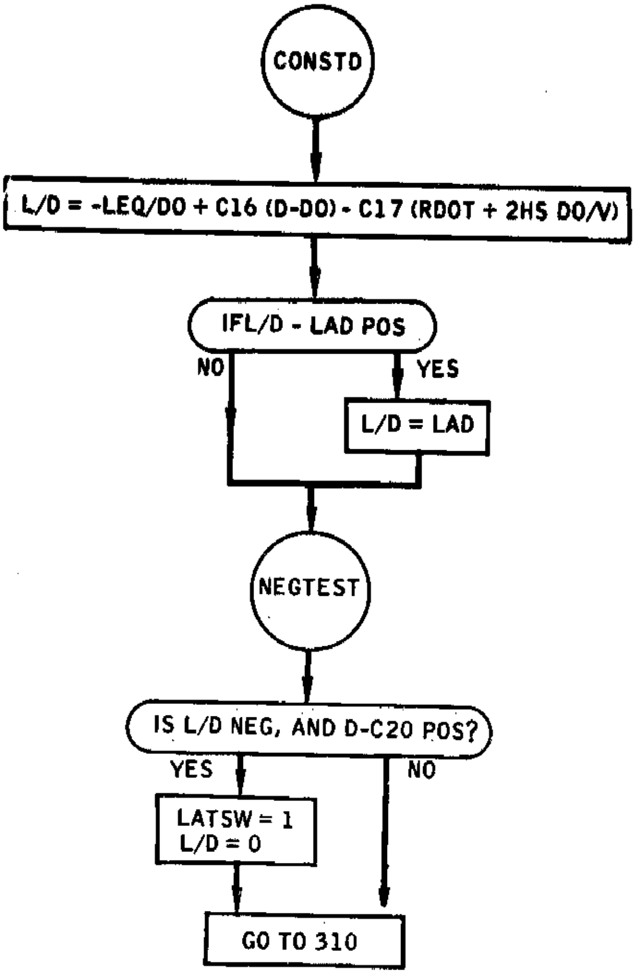

FIGURE 8.- CONSTANT DRAG

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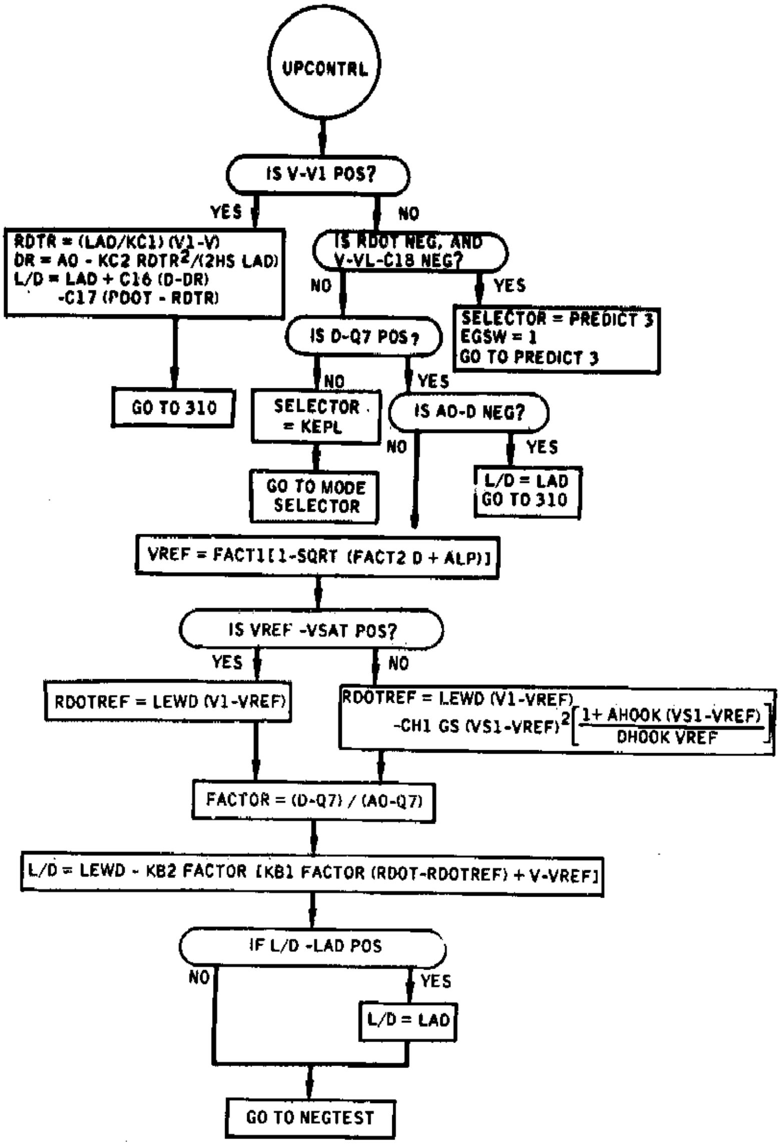

FIGURE 9.- UPCONTROL

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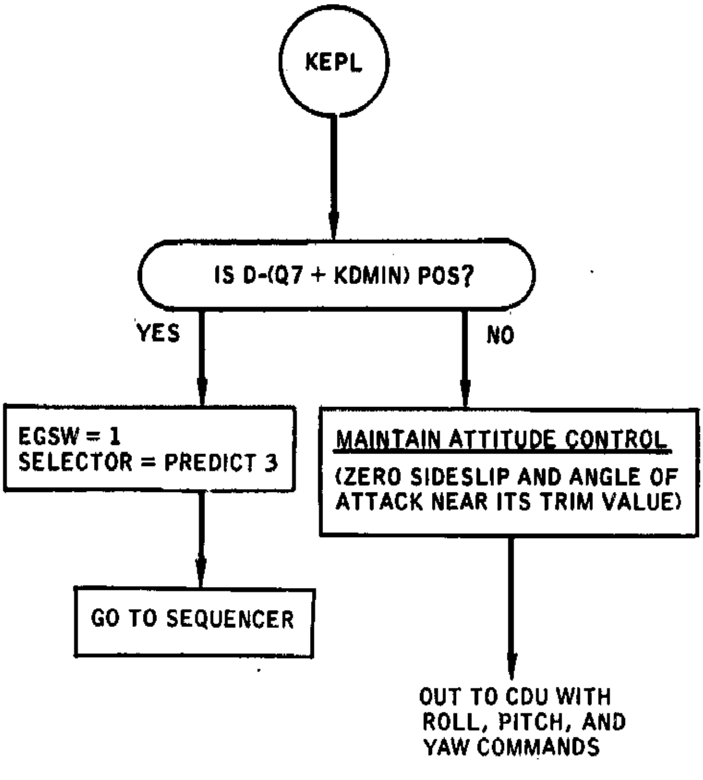

FIGURE 10.- KEPLER

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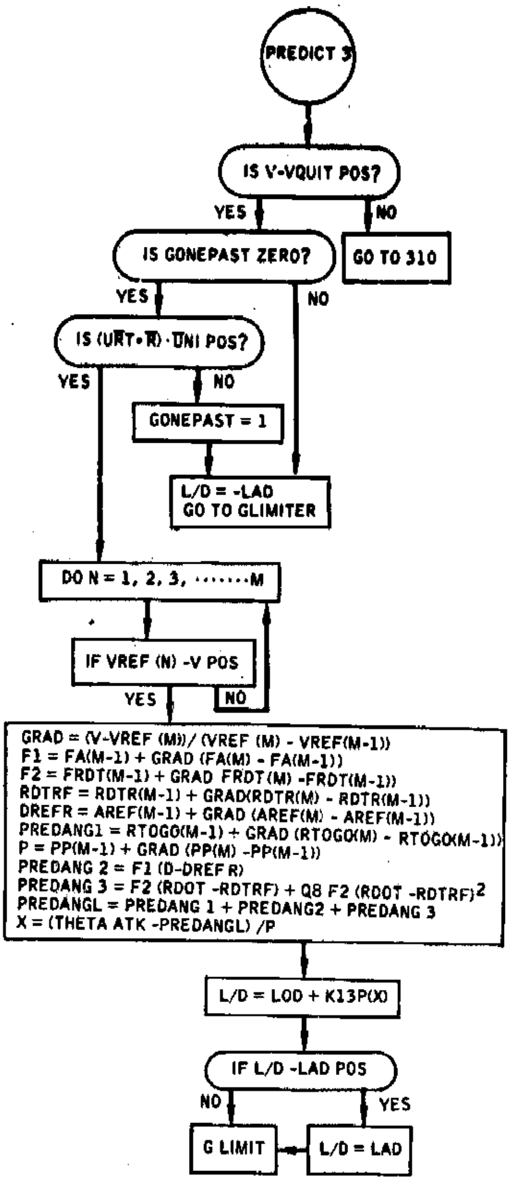

FIGURE 11.- PREDICT 3

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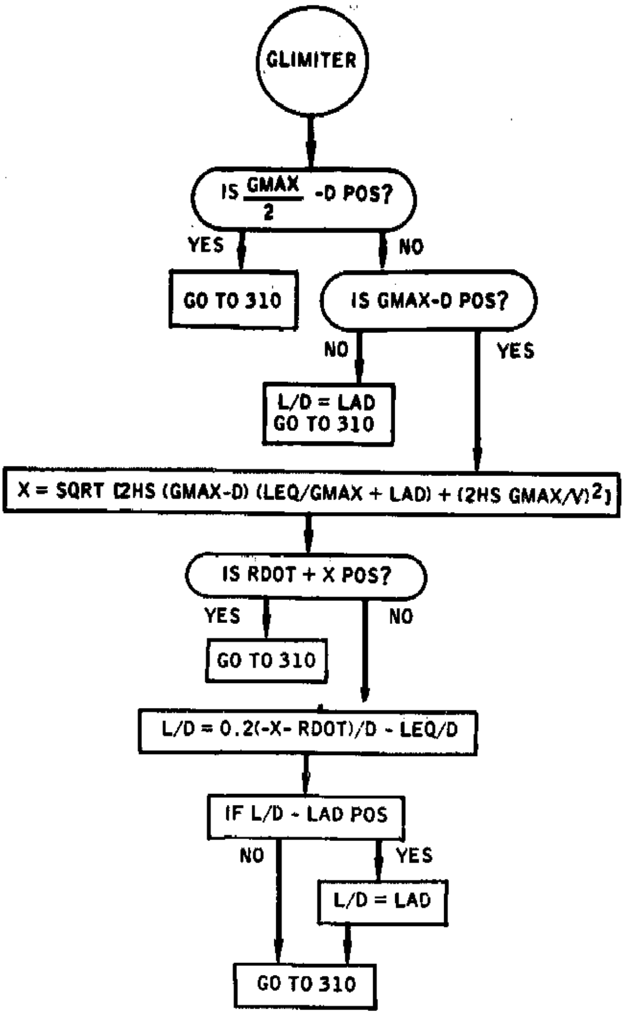

FIGURE 12.- G LIMITER

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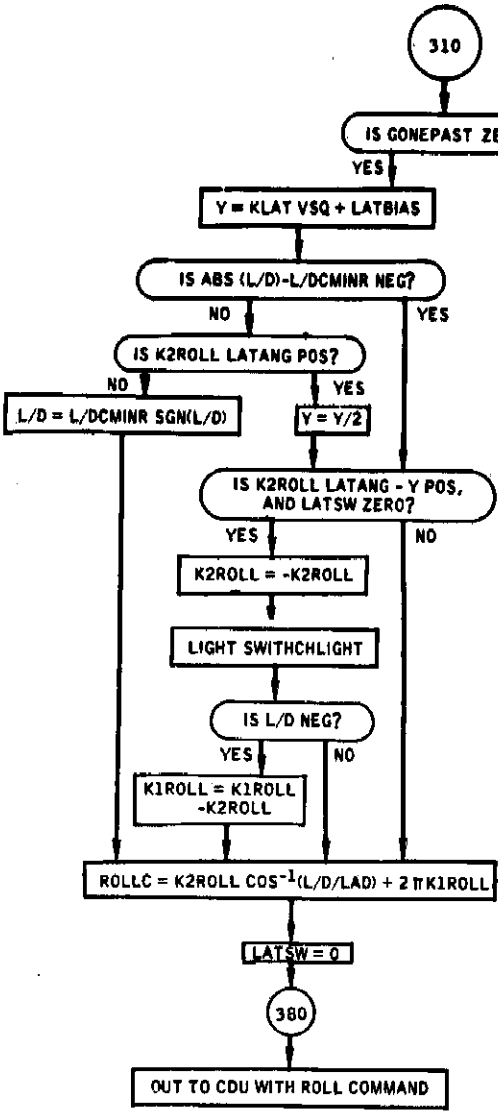

FIGURE 13.- LATERAL CONTROL

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